In compliance with the
Canadian Privacy Legislation
some supporting forms
may have been removed from
this dissertation.

While these forms may be included
in the document page count,
their removal does not represent
any loss of content from the dissertation.

University of Alberta

When will I use this? – Students' attitudes toward high school mathematics

by

Shauna Boyce

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of *Master of Education*

Department of *Secondary Education*

Edmonton, Alberta
Fall 2003

National Library of Canada
Acquisitions and Bibliographic Services
395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale du Canada
Acquisisitons et services bibliographiques
395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence
ISBN: 0-612-87601-2
Our file Notre référence
ISBN: 0-612-87601-2

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou aturement reproduits sans son autorisation.

Canada

University of Alberta

Library Release Form

Name of Author: *Shauna Boyce*

Title of Thesis: *When will I use this? Students' attitudes toward high school mathematics*

Degree: *Master of Education*

Year this Degree Granted: *2003*

Permission is hereby granted to the University of Alberta Library to reproduce single copies of this thesis and to lend or sell such copies for private, scholarly or scientific research purposes only.

The author reserves all other publication and other rights in association with the copyright in the thesis, and except as herein before provided, neither the thesis nor any substantial portion thereof may be printed or otherwise reproduced in any material form whatever without the author's prior written permission.

Signature

July 9, 2003

University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **When will I use this? – Students' attitudes toward high school mathematics** submitted by **Shauna Boyce** in partial fulfillment of the requirements for the degree of **Master of Education.**

Dr. Elaine Simmt

Dr. George Buck

Dr. Katherine Willson

Date: June 26 /03

ABSTRACT

This study investigated the attitudes toward, and beliefs about mathematics that were expressed by students enrolled in Mathematics 33 and Applied Mathematics 30. These two courses were developed for the mid-range high school mathematics student who does not intend to enter a mathematics-intensive faculty at a post-secondary institution. The students involved in the study were in grade 12, and between the ages of 16 and 18.

Based on an 15-question survey, observations, and selected interviews, the researcher determined that the AM 30 students were more able to see the purpose of studying mathematics than their counterparts in M 33, as they could see the relevance of it to more aspects of society, although neither class could see a great deal of relevance of mathematics other than to financial situations to their own personal lives. AM 30 students enjoyed mathematics class more than M 33 students. M33 students recognized mathematics in other classes more readily.

To my husband Geoff Beckett
whose unwavering support made
this accomplishment possible.
Thank you.

TABLE OF CONTENTS

Abstract *1*

Table of Contents *2*

Table of Abbreviations *5*

Chapter I: My Journey *1*
- How did I get here? 1
- The Programs 4
  - Mathematics 33 4
  - Applied Mathematics 30 5
- The Purpose of the Mathematics Programs 6
  - Mathematics 33 7
  - Applied Mathematics 30 8
- Significance of the Study 10
- Purpose of the Study 11
- Research Questions 12

Chapter II: An Academic Perspective *13*
- Learning and Knowing Mathematics – Complex Learning Theories 14
  - A constructivist view of knowing and learning mathematics 14
  - An enactivist view of learning and knowing mathematics 22
  - Complex learning theories and the classroom 25
- The World Outside of School – Mathematical Literacy 27
- Useful Mathematics 33
- Students' Attitudes Toward Mathematics 36

Chapter III: Methodology *42*
- Research Design 42
- Research Methodology 43
  - Qualitative research 43
  - Case study research 44
  - Reliability and validity 45
- Context of the Study 46
  - The jurisdiction 46
  - The schools 47
  - The teachers 47
  - The students 48
- Data Collection 49
  - Observational tools 49
  - Questionnaire 50
  - Interviews 52
- Data Analysis 53
- Delimitations 54
- Limitations 55

Chapter IV: Listening to Mathematics 33 Students *57*

Mrs. Stewart and her Classroom 57
What the Students Said 58
Enjoyment and interest in mathematics 58
How hard do I work? How much ability do I have? 63
Why am I in this class? 67
Will I take another math course? 68
Participation, Difficulty, and the World Outside of School 68
Active participation 69
Difficulty of the material 69
The world outside of school 69
Importance and Relevance of Mathematics 70
School mathematics and daily life 71
What the comments reveal 72

Chapter V: Listening to Applied Mathematics 30 Students ***74***
Mr. Rawdly and his Classroom 74
What the students said 75
Enjoyment and interest in mathematics 75
How hard do I work? How much ability do I have? 78
Why am I in this class? 84
Will I take another math course? 84
A positive relationship with the teacher, and active participation in class 85
Relationship with a teacher 85
Active participation 85
Importance and Relevance of Mathematics 86
The World Outside of School 88
What the Comments Reveal 88

Chapter VI: Putting it into Perspective ***90***
What the students said 90
Enjoyment of mathematics class 90
The world outside of school 92
Success in mathematics 93
Relationship with the teacher 95

Chapter VII: Conclusions ***98***
Important, Relevant, and Significant Findings 100
The Purpose of High School Mathematics 101
The role of the universities, colleges, and technical schools 103
Implications for Schools and School Jurisdictions 105
The role of the jurisdiction 105
The role of the educator 107
Recommendations 111
Implications for Further Study 112
Final Thoughts 113

References ***115***

Appendix: Questionnaire ***124***

Table of Figures

Figure 1: Alberta High School Mathematics Courses 3

Figure 2: Math 33 Students' Enjoyment Rating 58

Figure 3: Math 33 Students' Work Habits 63

Figure 4: Math 33 Students' Perceived Ability 64

Figure 5: Applied Math 30 Students' Enjoyment Level 76

Figure 6: Applied Math 30 Students' Work Effort 79

Figure 7: Applied Math 30 Students' Mathematical Ability 80

TABLE OF ABBREVIATIONS

Applied Mathematics 10	AM 10
Applied Mathematics 20	AM 20
Applied Mathematics 30	AM 30
Pure Mathematics 10	PM 10
Pure Mathematics 20	PM 20
Pure Mathematics 30	PM 30
Mathematics 33	M 33

CHAPTER I: MY JOURNEY

How did I get here?

I am a high school mathematics teacher, and have been one for thirteen years. I work for a rural school jurisdiction that borders a large urban area. I have taught high school mathematics at both of the high schools in this jurisdiction, and for a period of four years, I was seconded to Alberta Learning as the Examination Manager for the AM 30 and M 33 provincial diploma examinations. As such, I became very involved in the provincial implementation and assessment of these two programs. I also came to know many teachers around Alberta that were involved with both programs.

During my time with Alberta Learning, the M 33 program was being phased out, and the AM 30 program was being implemented. This was a tumultuous time for Alberta high school mathematics. Public opinion of the two new mathematics courses being implemented into Alberta high schools was less than favourable. Many teachers, students and parents believed the pure mathematics stream (PM 10-20-30) to be too rigorous, and the applied mathematics stream (AM 10-20-30) not to be rigorous enough.

The applied mathematics program had a particularly problematic introduction into Alberta schools. It seemed that one thing after another went wrong. First, there was no pilot year, resulting in schools offering the program for the first time without adequate resources. Second, the program proved to be so drastically different from what had previously been seen in Alberta high schools, that many teachers were intimidated by it from the start. Third, post-secondary institutions in Alberta expressed concern with its content, and acceptance of the program for entrance requirements became problematic.

As a *band-aid* solution, the provincial implementation of applied mathematics was twice delayed – first for one year, and then for a second. Schools had the option of implementing the program along with the pure mathematics program, or continuing to offer the Mathematics 13-23-33 stream. All of this of course, was played out in the media, staff rooms, and school councils around Alberta (McCabe, 2000).

However, underneath all of the public and political noise was a very quiet minority of teachers who had chosen to offer the applied program during the years of optional implementation. Many of these teachers were not only enjoying teaching the program, but were finding that their students were enjoying learning mathematics as well. These teachers often spoke to me about how their students were actually enjoying mathematics class, and answering the question "when will we ever need this?" instead of asking it. This stark contrast to the public profile that this program had been given intrigued me. How could these two perceptions be so drastically different?

As the manager of the diploma exams in both subjects, I was directly affected by the implications of students' and teachers' beliefs about mathematics in general, and about these courses in particular. As one who was intimately involved with both programs however, I became quite interested in the difference in philosophy and pedagogical underpinnings between them. Although my original interest for a research topic for this thesis centred on the relationship between instruction and the diploma examination, I began to realize that I was more interested in whether or not what these teachers were telling me could be demonstrated in students' opinions and attitudes. Could research demonstrate a link between attitudes, beliefs, and perceived usefulness of mathematics? I decided to investigate attitudes toward mathematics through the use of a

questionnaire in which questions relating to the perceived usefulness of mathematics to students and to society in general would be asked. I was curious to see whether or not this new approach to teaching and learning mathematics was indeed improving the outlook of students toward mathematics, increasing access to mathematics for all students, and increasing adolescents' enjoyment of learning mathematics.

Alberta high school students have a number of mathematics courses from which to choose, as can be seen in the chart below.

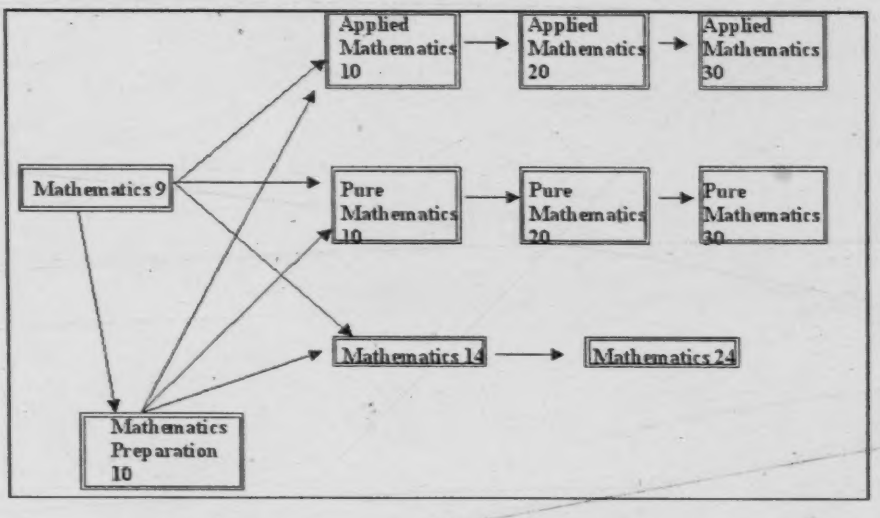


Figure 1: Alberta High School Mathematics Courses

When they enter high school, students can elect to enter the applied stream, or the pure stream if they have successfully completed grade 9 mathematics. If they have not successfully completed grade 9 mathematics, or if their prerequisite skills are weaker than desired, they can elect to enroll in Mathematics Preparation 10, which will help them to gain the prerequisite skills needed to be successful in either PM 10 or AM 10. Mathematics 14 and Mathematics 24 are courses designed to provide students with the basic mathematics skills required for the world of work and citizenship. These two

courses provide the minimum mathematics requirements for a high school diploma (Alberta Learning, 2002c).

Before the introduction of pure mathematics and applied mathematics as the two main streams in high school mathematics, students could choose from Mathematics 10, 20, 30 or Mathematics 13, 23, 33. The applied and pure courses were not intended to replace the original courses, but rather were designed to be different approaches to mathematical learning, but still of equal rigor.

The Programs

Mathematics 33

The M 33 program has been taught in Alberta schools since 1991. The school year 2002-2003 was the last year that this course was offered in schools. M 33 is intended to be the "mainstream mathematics program designed for students who require mathematics to prepare them for many post-secondary programs at universities, colleges, trades and employment" (Alberta Education, 1991).

The Mathematics 13-23-33 sequence emphasizes an inductive development of topics in geometry, algebra, trigonometry and statistics. Many of the topics are common to the Mathematics 10-20-30 sequence. The Math 13-23-33 sequence is based on the assumption that students need an expanded set of fundamental concepts, but also need to understand the ideas that make up those concepts and how they are related. "Most important, students must be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge" (Alberta Education, 1991, p.

1).

Although the intent was otherwise, M 33 was considered to be insufficient for admission into many post-secondary institutions in Alberta. Its acceptance did not reach nearly the same levels as the Mathematics 30 program also offered in high schools.

Applied Mathematics 30

In December 1993, the ministers of Education in British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory and Northwest Territories signed a document entitled the *Western Canadian Protocol (WCP) for Collaboration in Basic Education Kindergarten to Grade 12*. In signing this document, the provinces and territories agreed to aspire to common educational goals and standards for education. It was from this document that the new mathematics programs for Alberta high schools, pure mathematics and applied mathematics, were born. Both of these programs were implemented at the grade 10 level in 1998, although AM10 endured two years of optional implementation before its compulsory implementation in September 2000.

The applied mathematics program focuses on the application of mathematics in problem solving. The approach to learning mathematics in this program is primarily data driven, and uses numerical and geometrical problem-solving techniques. Relevance for students is a key issue in this program, and to increase this, students collect data in experiments and activities, and analyze the raw data to develop mathematical concepts. Algebraic constructs are addressed as a result of activities or experiences rather than a means to activities and experiences (Alberta Education, 1998). The key difference between AM 30 and M 33 is a belief in the kinds of mathematics that are deemed important. M 33 is grounded in the belief that students need to understand traditional

concepts such as trigonometry, geometry and algebra, and takes an algebraic approach to learning mathematics. The applied mathematics program is grounded in the belief that a numerical approach to understanding how mathematics is used is more beneficial to students. Traditional topics such as trigonometry and geometry are still studied, but other concepts such as matrices and fractals are also considered. Algebra is studied as required for practical applications, rather than for its own sake.

The applied mathematics program is intended to give students a clearer picture of why they are learning mathematics and thus, to motivate them in this learning. The goal is to have students "experience mathematics as being dynamic and useful in their careers and everyday life" (Alberta Education, 1998, p. 1). My research should test this goal by comparing AM 30 students' beliefs about mathematics and learning mathematics with their counterparts in M 33.

AM 30 received limited acceptance as a prerequisite for admission to post-secondary schools, however it is more widely accepted than M 33. This acceptance however, took over three years for Alberta Learning to negotiate, and it by no means reaches the same level of acceptance as the more theoretical stream of mathematics (PM 30) offered in Alberta schools.

The Purpose of the Mathematics Programs

M 33 and AM 30 are courses that were designed to serve the same purpose – offering students who are not bound for a post-secondary education in a mathematics or science-related field a course that would serve them well in their lives beyond high school. However, the view of topics and approaches considered important are

significantly different between the two programs.

Mathematics 33

When M 33 was introduced to Alberta schools, the view of mathematics in Alberta Education's eye had shifted. No longer was the emphasis on procedures, nor was it seen as important for students to memorize mathematical formulas and algorithms. Now the move was toward a more dynamic view of mathematics as a "precise language, used to reason, interpret and explore" (Alberta Education, 1991, p. 1). The program developers tried to incorporate these beliefs into the outcomes of the program of studies, and thus promoted the study of such topics as quadratic functions, trigonometry, and radical numbers. There was also an emphasis on applications of mathematics to a world outside of school, as can be seen by the inclusion of units of study such as annuities, mortgages and loans.

Within the *Program of Studies*, the suggestions for instructional style and teaching methods to promote the intended philosophy were implicit, but no direct statements were made. The recommendation was to combine more traditional problem-solving techniques with ones such as estimation and simulation, and the use of modern technology to solve problems to which the mathematics could be applied (Alberta Education, 1991). Alberta Education promoted the use of technology and problem-solving strategies in the teaching of mathematics, but did not direct teachers to approach topics in any particular manner. The *Program of Studies* suggested that content be learned in a problem-solving context to promote mathematical literacy, which Alberta Education defined as referring to students' ability and inclination to "manage the demands of their world through the use of mathematical concepts and procedures to

communicate, reason and solve problems" (Alberta Education, 1991, p. 3).

By the time a diploma examination was introduced into the M 33 course in 1996, there had been a great deal of research done on learning mathematics that indicated that students needed to construct their own understanding of mathematics and the importance of connecting this knowledge to the world outside of school (Steen & Forman, 1995; NCTM, 1991). The team of test-developers responsible for the provincial examination was aware of this research and tried to incorporate applications from the world outside of school into the diploma examination. Questions on the diploma examination were designed around scenarios coming from Alberta industry, consumerism, sports, and many financial situations (Alberta Learning, 2002a).

Applied Mathematics 30

When the prescribed curriculum changed to include AM 30, the emphasis on the uses and purposes of mathematics and mathematics education had shifted once again. Educators, and society in general, were being told of the need for Canada to be competitive in the global economy, and the implications of competitiveness included increased emphasis on mathematics, science and technology education. The effect of this is seen in curriculum changes in classrooms across the country. In most regions of Canada, instead of arithmetic, writing, and reading, mathematics and science, technology, literacy, and communication skills have become the core of study.

As part of this movement, the mathematics curricula in Alberta were drastically revised. Teaching mathematics was reconceived as the provision of activities designed to encourage and facilitate the constructivist pedagogy. The mathematics classroom was to be a community of inquiry. Problem posing and problem solving, and developing an

approach to thinking about mathematics would be more highly valued than memorizing algorithms and simply looking for right answers (Shifter, 1996). Process, rather than product was viewed as the most important aspect of learning and doing mathematics.

The National Council of Teachers of Mathematics (NCTM) has recently produced its *Principles and Standards for School Mathematics* (NCTM, 2000). Amongst its principles for school mathematics, one finds statements such as "students learn by connecting new ideas to prior knowledge" (p. 18) and "learning with understanding is essential to enable students to use what they learn to solve the new kinds of problems they will inevitably face in the future" (p. 21).

Although the WCP was developed before the publication of *Principles and Standards*, (2000) the project is still related, as the *Principles* were meant to be an extension of the 1989 *Standards* published by NCTM. These standards are the basis on which the *WCP* (and therefore the Alberta programs of studies) was developed. The structure of these documents reflects constructivist pedagogy as can be seen by some of the belief statements that very closely resemble those of the NCTM.

> Students are curious, active learners who have individual interests, abilities, and needs. They come to classrooms with different knowledge, life experience, and backgrounds that generate a range of attitudes about mathematics and life. Students learn by attaching meaning to what they do; and therefore, they must be able to construct their own meaning of mathematics. (Alberta Education, 1996, p. 2)

Constructivist pedagogy suggests that the teacher should value and respect each student's way of thinking, so that the learner feels comfortable in taking intellectual risks,

asking questions, and posing conjectures. A constructivist classroom is one in which problem solving, and the use of projects and research to learn mathematics is encouraged.

Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. A greater proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfillment *(WCP,* 1996).

The other difference in the pedagogy of the new mathematics programs was an emphasis on changing students' attitudes toward mathematics. For the first time, programs of study in Alberta contained statements such as "It is important for students to develop a positive attitude toward mathematics so that they can become confident in their ability to undertake the problems of a changing world, thereby experiencing the power and usefulness of mathematics" (Alberta Education, 1998, p. 2).

Significance of the Study

We as mathematics teachers can no longer continue to believe that mathematics learning is independent of who is learning and the environment in which it is taught. In fact, what we teach, and how we teach it are greatly affected by students' attitudes toward the subject and the topics within it, and their beliefs about the usefulness and importance of mathematics.

Regardless of the prescribed curriculum in Alberta high schools, the impact on learning is, and will continue to be minimal without considering the learners' attitudes toward the content. Attitudes play a critical role in establishing and maintaining an

interest in subject matter. This study then, illustrates its significance by concerning itself with the interest levels of students, their existing and evolving beliefs about mathematics, and their perceptions of what purpose it serves for them. If the study can inform practice for teachers and curriculum developers, it may be able to shape the attitudes of students.

Purpose of the Study

The purpose of this study was to listen to the voices of the students to whom we teach mathematics, with respect to their beliefs about, and attitudes toward mathematics, and to better understand their experiences in AM 30 and the M 33, given the differences in curriculum and approaches to learning. Reform in mathematics education in Alberta was subjected to the voices of politicians, bureaucrats, teachers, and parents, but the voice of the student was rarely heard.

The NCTM emphasizes the importance of promoting positive attitudes toward, and interest in mathematics. Without a general interest in, and positive attitudes towards mathematics, a serious problem could emerge for a society that increasingly depends on a quantitatively literate citizenry (NCTM, 2000).

The delay in the implementation of the new programs of study for the math courses in Alberta was unfortunate, and caused an uncomfortable environment for many teachers, students, and parents, but those same difficulties provided me with a unique opportunity to study the attitudes of students who are registered in the two distinct programs. The purpose of my research was to examine the experiences and attitudes toward mathematics that students in these two courses exhibit.

There has been a significant amount of research done in the area of beliefs and attitudes toward mathematics, but much of it is related to students with special needs

(Montague, 1997); has a focus on gender differences (Steinback & Gwizdala, 1995); or has been an investigation into the immediate effects of introducing a new type of teaching style or tool into the classroom (Higgins, 1997; Funkhouser, 1993). Although, there have been a few studies (Ma & Kishor, 1997; Boaler, 1998) not very many have studied attitudes of high school, mainstream mathematics students.

Research Questions

The research considers the question **In what ways do students' attitudes and experiences compare, given the contexts of Applied Mathematics 30 and Mathematics 33?**

More specifically, some of the sub-questions are

- What importance and relevance do students place on mathematics?
- What aspects of math class do students enjoy or not enjoy?
- To what do students attribute their success in mathematics?
- Do students consider mathematics important to society or to their lives?
- Do students recognize where they use, or could use the mathematics that they have studied?

CHAPTER II: AN ACADEMIC PERSPECTIVE

How do students' attitudes and perceptions of mathematics compare, in the context of AM 30 and M 33? This question needs to be informed by research into learning and knowing mathematics, the importance of mathematics that relates to the world outside of school, and how mathematics that is considered useful by students affects their learning. All of these contribute toward an exploration of students' attitudes toward mathematics. The following chapter explores the relevant literature surrounding these topics.

A review of the literature surrounding the current learning theories is important because, if I as a researcher intend to understand how and why the students involved in my research related to particular topics that they studied, and not to others, I must first develop an understanding of how students learn and come to know mathematics. This may help to reveal which aspects of the students' comments can reveal important information about their learning.

Both of the courses in which the participants in this study are enrolled have been designed with a conscientious effort to relate mathematical topics to the world outside of school, thus an investigation into the importance of this relationship is required. The assumption is that if students can see how their understandings of mathematics relate to their current lives, and the lives that are in their future, they will come to understand how important the study of mathematics is. By relating mathematics to the world outside of school, curriculum designers intended to develop mathematically literate citizens. The research has much to say about mathematical literacy and this needs to be explored if one

wishes to understand the intent of the curriculum designers.

Finally, the purpose of this study was to better understand the experiences of AM 30 and M 33 students, by examining their attitudes toward, and beliefs about high school mathematics. Informing a discussion of how students develop positive or negative attitudes towards learning and doing mathematics involves investigating other studies that were related to similar topics. These studies provide insight into what defines attitude, how attitude relates to learning, and how attitudes toward mathematics tend to change as students progress through school.

Learning and Knowing Mathematics – Complex Learning Theories

Radical constructivism, social constructivism, and enactivism are among the most commonly discussed learning theories surrounding mathematics education. Each one regards the learner in complex terms and moves beyond the behaviorist and mentalist theories that separate mind and body to be more mindful of the sorts of actions that those two theories take for granted. Both constructivism and enactivism recognize thinking as being *part* of the universe instead of about it. For theorists sympathetic to these points of view, "learning is ... understood as a participation in the world, a co-evolution of knower and known that transforms both" (Davis, Sumara, Luce-Kapler, 2000, p. 64).

A constructivist view of knowing and learning mathematics

Constructivism is most often articulated in contrast to behaviorism and mentalism. Both behaviorists and mentalists emphasize observable, external behaviours, and as such, centre on learners' efforts to accumulate knowledge about the world, and teachers' efforts to transmit that knowledge. Knowledge represents the real world,

considered separate and independent of the knower. This knowledge is considered true if it correctly reflects or models that independent world (Steffe & Kieren, 1994; Davis, Sumara, Luce-Kapler, 2000).

Constructivism stems from a transition from a Cartesian, endogenic view of knowledge to a more exogenic view; from a static, passive view to a more adaptive and active view (Davis, Sumara, Luce-Kapler, 2000; Steffe &Kieren, 1994). This theory holds that knowledge and reality do not have an objective value. Von Glasersfeld (1995a) indicates that reality is made up of things and relationships that we rely on, and on which, we believe, others rely on as well. The knower constructs (or construes) his/her own reality based on personal experiences and interactions with the environment. The definition of truth then, is not a match to reality, but rather is a fit with the environment. From a constructivist point of view, "adaptation does not mean adequation to an external world of existing things-in-themselves, but rather improving the organism's equilibrium, i.e., its fit, relative to experienced constraints" (von Glasersfeld, 1995c, p. 63). The learner's basis of meaning is found in her or his direct experience with a dynamic and responsive world (Davis, Sumara, & Luce-Kapler, 2000, p. 65).

There are many types of constructivism, but it is radical constructivism and social constructivism that are commonly used in mathematics education research. As well, the two programs in which the students in my research are enrolled were designed with a constructivist framework. Hence, these two theories will be discussed in detail.

Radical constructivism.

Radical constructivists are concerned with the individual's experiences and construction of reality, rather than the question of the nature of reality. The measure of

truth for a radical constructivist is not empirical evidence, but rather a successful action. The definition of wrong, then, is simply an unsuccessful action. Radical constructivists assume that an individual makes sense of experience in order to satisfy an essential need to obtain predictability and control over one's environment (Confrey, 1994).

According to von Glasersfeld (1990), there are two basic principles of radical constructivism. First, knowledge is not passively received either through the senses or by way of communication, but rather, the cognizing subject actively builds up knowledge. Second, the function of cognition is adaptive, (in a biological sense), tending toward fit or viability. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. Viable schemas are changed through processes of assimilation or accommodation as new circumstances are met. Von Glasersfeld interprets assimilation to mean that new material is treated as an instance of something known. "The cognitive organism perceives (assimilates) only what it can fit into the structures it already has . . . [and] remains unaware of, or disregards, whatever does not fit into the conceptual structures it possesses" (von Glasersfeld, 1995b, p. 63). The recognition of a certain situation is a result of assimilation. A certain activity associated with that situation produces a result that the student will attempt to assimilate to his or her expected result. If the organism is unable to do this, there will be a perturbation.

> The perturbation, which may be either disappointment or surprise, may lead to all sorts of random reactions, but one among them seems particularly likely: if the initial situation is still retrievable, it may now be reviewed, not as a compound triggering situation, but as a collection of

> sensory elements. This review may reveal characteristics that were disregarded by assimilation. If the unexpected outcome of the activity was disappointing, one or more of the newly noticed characteristics may effect a change in the recognition pattern and thus in the conditions that will trigger the activity in the future. Alternatively, if the unexpected outcome was pleasant or interesting, a new recognition pattern may be formed to include the new characteristic, and this will constitute a new scheme. In both cases there would be an act of learning and we would speak of an accommodation. (von Glasersfeld, 1995b, pp. 65-66)

Thus, the learning theory that emerges from Piaget's work can be summarized by saying that cognitive change and leaning in a specific direction occur when a scheme, instead of producing the expected result, leads to perturbation, and perturbation in turn, to an accommodation that will either maintain or re-establish equilibrium (von Glasersfeld, 1995b). Understanding then, requires a pupil to reflect upon these perturbations and his or her reaction to them. Understanding only occurs after reflection, and this is an activity that students must carry out themselves (von Glasersfeld, 1995c). It is worth noting that von Glasersfeld defines understanding much like Skemp (1987) defines relational understanding – "knowing both what to do, and why" (p. 153).

Cognitive development is most likely to occur in situations where it can be related to actual experiences and it leads to a solution to a problem. Because people learn new knowledge by building on existing knowledge, subject matter must relate to everyday experiences. "In mathematics the *reflective process*, wherein a construct becomes the *object* of scrutiny itself, is essential (Confrey, 1990, p. 109, emphasis original), not

because mathematics is removed from everyday experience but rather because it is built from human activity (counting, folding, ordering, and comparing). Mathematics is a language of human action.

> This is yet another confirmation of the general principle that intellectual development, although manifested in the increased ability to think in abstraction from sensorimotor material, is not only triggered by interactions with such material, but requires, even at its highest levels, at least sporadic returns to the level of concrete experience. One reason for these returns is that, no matter how far a conceptual structure has been removed from sensorimotor experience, its ultimate viability can be confirmed only on that level. (von Glasersfeld, 1995c, pp. 381-382)

The focus of radical constructivism is on individual cognition. Therefore, the learner is considered to be an autonomous, self-contained, and self-determining form. Learning mathematics then, is a matter of interaction with the mathematics. The teacher can help students to learn by providing occasions for their mathematical activity, but it is the student's way of making sense that determines their own knowledge (Steffe & Kieren, 1994).

Teachers working within a constructivist paradigm both model and elicit mathematics, but they model by asking questions, following leads, and conjecturing rather than presenting facts and products. "Teachers must never be seen, nor indeed ever see themselves, as mechanics of knowledge transfer. Rather, they should feel and act as the intuitive helpers who, in Socrates' words play the role of the midwife in the birth of understanding" (von Glasersfeld, 1995c, p. 383). Teaching this way requires

considerable mathematical knowledge and pedagogical skill (Noddings, 1990).

Radical constructivism speaks to the need to consider students' attitudes and emotions when learning mathematics through its discussion of reflection on perturbations and reactions to them, but it lacks a prescriptive approach to education. Rather, radical constructivists concentrate on offering descriptions of the complex dynamics of an individual's learning. To consider more of an approach to classroom dynamics and environment, we may turn toward social constructivism.

Social constructivism.

In contrast to radical constructivism, social constructivism focuses more on the social body than the individual. Social constructivists are interested in the group interactions surrounding a learner as (s)he builds understandings and comes to share conclusions with the group. The focus is on "...conversation patterns, relational dynamics, and collective characters. Cognition ... is always collective: embedded in, enabled by, and constrained by the social phenomenon of language, caught up in layers of history and tradition; confined by well-established boundaries of acceptability" (Davis, Sumara, & Luce-Kapler, 2000, p. 67). Social constructivists "focus on the processes by which students learn as they reciprocally adapt to each others' and the teacher's mathematical activity, thereby establishing taken-as-shared mathematical meanings" (Cobb, Yackel, & Wood, 1992, p. 18).

Successful communication requires that individual interpretations are compatible, or fit the purposes at hand; that is, possible discrepancies do not become apparent in the course of ongoing social interactions. "The distinction is not between students' internal representations and mathematical relations contained in external representations but

between the students' interpretations of instructional materials and the taken-as-shared interpretations of mathematically acculturated members of the wider community" (Cobb, Yackel & Wood, 1992, p. 17).

Social constructivists understand objective knowledge of mathematics to be social, and not contained in texts or other recorded materials, or in some ideal realm. Objective knowledge of mathematics resides in the shared rules, conventions, understandings and meanings of the individual members of the mathematics community, and in their interactions (Ernest, 1991). Thus, the role of the teacher can never be to impart knowledge, but rather can only represent knowledge in the public domain (an intermediate step between subjective and objective knowledge) and provide the opportunity for students to develop their own subjective knowledge. Once subjective knowledge from the group is publicly represented, the path to objective knowledge through social acceptance can begin.

To know mathematics from a social constructivist perspective is to maintain the relationship between subjective and objective knowledge. These realms are "mutually dependent" and serve to "recreate each other" (Ernest, 1991, p. 81). Ernest believes that objective mathematical knowledge is reconstructed as subjective knowledge by the individual, through interactions with others, and by interpreting texts and other inanimate sources. These interactions provide, especially through negative feedback, the means for developing a fit between an individual's subjective knowledge of mathematics, and socially accepted, objective mathematics (Ernest, 1991). Hence, social interaction within a classroom, both between students and students and teachers, is of utmost importance. This perspective goes a long way to suggest reasons why traditional methods of lectures

and worksheets do not work very well to support the learning of mathematics. It also supports Bauersfeld (1995), who states that there is no such thing as discovery learning, at least in the sense of revealing or unmasking of any kind of external structures.

Cobb, Yackel, and Wood (1992) suggest that activities that are starting points for students' mathematical constructions should draw on students' prior experiences when guiding the negotiation of initial conventions of interpretation and students' interpretations and actions in these situations should produce highly situated, intuitive bases from which they abstract as they construct increasingly sophisticated mathematical conceptions. Steen (1989) also suggests that these classroom practices must emphasize active learning, problem solving, concrete materials, and continual assessment.

Social constructivism then would speak to the need to consider students' attitudes and beliefs while learning mathematics. When designing mathematics lessons, teachers must relate the mathematics to students' experiences. Providing such a foothold, on which students can build their understanding, is imperative. These students then, are able to relate the mathematics they are learning to other, personal situations. This in turn allows them to relate more readily to the mathematics – something that helps to alleviate the fears and misconceptions that many students have about mathematics.

Constructivism however, (both radical and social) is not without critics. One of the most often heard criticisms is concerned with its delimitations – its narrow scope, especially in terms of instruction. Kieren (1998) states that if one sees learning through a constructivist view, the acts of the teacher become part of the *background* of learning. While constructivism has provided a basis for insight into individual cognition, it is an inadequate framework for the re-interpretation of teaching. Constructivism fails to enable

us to address issues such as our interactive capacities or complex social situations. As such, it is of limited value (Davis, Kieren, & Sumara, 1996).

Another criticism of constructivism is that it is overly relativistic. If everyone is bound by his or her own constructs, and if no appeal to an external reality can be made to assess the quality of those constructs, then everyone's constructs must be equally valid. As Davis, Kieren, & Sumara (1996) assert, constructivists only consider interactional dynamics with the environment from the point of view of the individual. The subjective-objective dichotomy suggested by social constructivists lacks adequate interpretive power to address the question of knowledge (Davis, 1995). To address this question then, one may move toward the theoretical framework of enactivism to gain a more ecological perspective.

An enactivist view of learning and knowing mathematics

Like constructivism, enactivists assume a logic that implies survival of the fit (rather than the fittest) – the survival of a truth is by adequacy rather than optimality. Learning then, rather than being a process of selecting correct actions, is one of discarding those actions that do not work (Davis, Kieren, & Sumara, 1996). Unlike constructivism however, "enactivism does not regard the individual cognizing agent as its object of study or as the location of cognition. Rather "enactivism argues for a conflating of such issues as individual learning and collective action" (Davis, 1997, p. 366).

Enactivists consider the question of the location of knowledge and the ultimate source of collective knowledge to be a non-issue that finds its roots in the modernist conception of identifying self as independent and isolated. The understanding of self

cannot be abstracted from the world that contains it, but instead should be considered to *be* the world. That is, knowing, being and doing are not three things, but rather, only one (Davis, Sumara, & Kieren, 1996). There is "a certain self-similarity (and nestedness) between the processes of individual cognition and of collective action. In this formulation, it makes little sense to study the emergence of an individual's understandings without considering the social and political contexts in which such understandings arise" (Davis, 1997, p. 374).

Enactivism emphasizes the complexity of learning. Instead of viewing the learner as coming to know, the learner and the learned, the knower and the known are co-evolving and being co-implicated.

> Learning should not be understood in terms of a sequence of actions, but in terms of an ongoing structural dance – a complex choreography – of events which, even in retrospect, cannot be fully disentangled and understood, let alone reproduced. (Davis, Kieren, & Sumara, 1996, p. 153)

Experiences and learning are not linear, but rather, they are transformative, complex, and somewhat unpredictable. They can be considered using the metaphor of a waterfall. It has mobile shape that is permanent, yet it never really stands in front of us. The composition of the waterfall and our experiences of it constantly change – only its form remains permanent. The waterfall, experiences, and learning all exhibit a *dynamic form* (Davis, Kieren, & Sumara, 1996).

Departing from conceptions of cognition as an internal, and largely invisible process, enactivists, (Varela, Thompson, & Rosch, 1991; Davis, Kieren, & Sumara, 1996; Maturana & Varela, 1987) argue that a fuller understanding of the phenomenon of

learning must embrace the dynamic and complex interplay of individual and environment. Knowledge cannot be separated from action. Essentially knowing is doing, which ultimately cannot be separated from an understanding of self identity. Knowledge is not uncovered or invented, but rather is enacted through the history of our participation in a dynamic and responsive world. Like the subject matter of a conversation, its nature, structure, and results can never be anticipated, let alone fixed (Davis, Kieren, & Sumara, 1996). To use a mathematical comparison, the relationship between our identities and the character of the collective is "not one of fragment to whole, but of fractal to complete image. The whole unfolds from the part, and is enfolded in the part" (Davis, 1995, p. 7). Thus,

> mathematics and mathematical understanding are collective phenomena, not individual ones. Even when an individual is working independently and in apparent isolation on a mathematical task...the action is social, for it is framed in language and procedures that have arisen in social activity. (Davis, 1995, p. 4)

Although studying the action of a particular learner may be informative, the focus of inquiries into the natures of knowing and knowledge must include the collective.

Kieren (1998) views mathematics as an activity that is fully determined by a person's structure, in which he or she "brings forth a world of mathematical significance" (p. 7). His claim is that this view of mathematics learning and knowing allows us to appreciate how mathematical activity in a classroom involves and enhances problem solving; and how the actions of the student and the teacher are complicit in the learning of others (i.e., many actions occurring together that influence and change every other

action in the 'world' [classroom] in which they occur). Every time a statement, question, or action is put forth, the world changes, and this in turn, changes one's perception of the world. Instead of viewing mathematical knowledge as unchanging, engaging in mathematical activity enacts a change in the character of mathematics (Kieren, 1998; Davis, 1995). This is not however, a simple back-and-forth activity, but rather one that occurs in a complex web, with each person's actions resulting in new possibilities for all others (Kieren, 1998). Mathematical knowledge then, is located in the "*inter*-activity of learners" (Davis, 1995, p. 4, emphasis added), involving both individual action as the heart of mathematical cognition, and the role of the environment, including the artifacts, actions, and persons within it (Kieren, 1998).

Complex learning theories and the classroom

Mathematics is subject to change. It is instilled with social, gender, and political ideals, and some of its conventions are quite arbitrary (Davis, 1997). Yet this notion goes virtually unconsidered among mathematics classroom teachers. For the most part, mathematical knowledge has been regarded as something *out there* – "objectively true, pre-existent, independent of human agency" (Davis, 1995, p. 3).

Perhaps the biggest impediment to the adoption of teaching styles that support complex learning theories lies in the fact that many teachers feel they give up control of the class if they give more responsibility to the students for their learning. When a teacher lectures she knows that the content has been covered because she has controlled its presentation. The notion of distributing control has rarely been considered.

The myth of coverage is one of the largest barriers to adoption of constructivist

pedagogy. Many teachers believe that if a mathematical topic is covered, students will learn it, and therefore feel pressured to cover huge amounts of material, necessarily at great speed. Most often, this leads to disappointing results.

> [B]ecause students in traditional curricula learn ideas and procedures by rote (if at all) rather than meaningfully, they quickly forget them, so the ideas must be re-taught year after year. In sense-making curricula, on the other hand, because students retain learned ideas for long periods of time, and because a natural part of sense making is to interrelate ideas, students accumulate an ever-increasing store of well-integrated knowledge. (Battista, 1999).

Panitz (1997) also suggests that students themselves may not enjoy cooperative learning techniques because they are unfamiliar with cooperation (they have been trained to *compete* in school) or because they are no longer able to remain passive. This may also serve as an impediment to implementing a different view of mathematics – it makes teachers uncomfortable, and students dislike it. But the traditional methods of instruction ignore students' personal construction of mathematical meaning, and as such, the development of their mathematical thought is not properly nurtured (Battista, 1999).

Finally, teachers may have a lack of awareness of issues surrounding education (Apple, 1992). To gain such awareness, teachers require a supportive community in which they can discuss the issues. During the discussions, topics such as how people learn, the nature of mathematics and the implications for teaching, and the purpose of formal education must be explored (Davis, 1997).

The World Outside of School – Mathematical Literacy

Some mathematicians and mathematics educators believe that, in essence, the relevance of mathematics to a student's world outside of their formal education is unimportant. The NCTM however, disagrees. The importance of mathematics that relates to the world outside of school is highlighted in the *Principles and Standards for School Mathematics*, published by the NCTM. Some of their belief-statements follow:

- Knowing mathematics can be personally satisfying and empowering. The underpinnings of everyday life are increasingly mathematical and technological.
- Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects.
- Just as the level of mathematics needed for intelligent citizenship has increased dramatically, so too has the level of mathematical thinking and problem solving needed in the workplace, in professional areas ranging from health care to graphic design.
- Although all careers require a foundation of mathematical knowledge, some are mathematics intensive. More students must pursue an educational path that will prepare them for lifelong work as mathematicians, statisticians, engineers and scientists (NCTM, 2000).

In making such suggestions, NCTM challenges the assumption that mathematics is only for the select few. Rather, understanding mathematics becomes essential for all students.

The provincial curriculum designers in Alberta have taken a stance similar to that

of the NCTM by creating programs of study for M 33 and AM 30 that relate to the world outside of school and by emphasizing everyday, contextual mathematical situations. In doing so, an attempt has been made to promote mathematical literacy. These curriculum designers would agree with Porter (1997) and Kolata (1997) who believe that mathematical literacy, or the lack thereof, has both civic and political implications. In a world that is becoming more and more dependent on, and full of mathematics, those who are functionally mathematically illiterate will be at a disadvantage. They will be less likely to secure and maintain good jobs, or to make informed political choices. This is not as simple as being unable to understand some information from newspapers and television, but also that press treatment of mathematical and quantitative issues can not be very probing if the audience does not understand how the numbers came about or what they mean (Porter, 1997; Kolata, 1997).

The test of mathematical literacy is whether a person naturally uses appropriate skills in many different contexts. Educators know all too well the common phenomenon of compartmentalization, when skills or ideas learned in one class are completely forgotten when they arise in a different context. "This is an especially acute problem for school mathematics, in which the disconnect from meaningful contexts creates in many students a stunning absence of common number sense" (Steen, 2001, p. 6). Research suggests that many students experience difficulty transferring and applying mathematical skills to new contexts, both within and outside of the mathematics classroom (Carraher, Schliemann, & Carraher, 1988; Ginsberg, & Gal, 1995; Singley, & Anderson, 1989).

Recognizing familiar mathematical concepts in another field (like a science class) requires an understanding of the context, and therefore, students must be encouraged to

view course material through a quantitative lens. This means that mathematics teachers must devote more time to developing students' insight in recognizing mathematical ideas in context (Hughes-Hallet, 2001) and less time with mathematics problems that are contrived or are presented out of context (Gal, 1997). Mathematical situations that arise outside of school are always embedded in a context that has some personal meaning to the people involved. It does not follow however, that teaching more mathematics necessarily leads to increased numeracy. Mathematical literacy is less about abstract concepts as it is about applying basic tools in sophisticated settings. Mathematical literacy, rather than calculus, is the key to understanding our data-drenched society (Steen, 2001).

The tradition of de-contextualized mathematics instruction has failed many students – those who leave school with neither the numeracy skills nor the quantitative confidence required in society (Steen, 2001). Many of these young people cannot practice mathematics after they finish high school mathematics (Paulos, 1988), and unfortunately, despite years of study and life experience in an environment immersed in data, many educated adults remain functionally mathematically illiterate (Steen, 2001).

There are many examples of students with sophisticated mathematics course work in their backgrounds who possess minimal quantitative literacy, as well as many examples of students with remarkable levels of quantitative literacy but little formal mathematics. This suggests that trying to improve quantitative literacy by requiring more mathematics courses is at best, inefficient (Hughes-Hallet, 2001, p. 94).

Ellis (2001) found that students leaving his elementary algebra course could solve fewer contextualized mathematical problems after they completed the course than they

could before they took it. He found that the students believed, after completing the course, that they had to use symbols to solve problems even though they previously were able to solve them with reasoning and arithmetic. Similarly, students in his differential equations course, although apparently quite adept at manipulating symbols, often could not interpret the results of their manipulations (Ellis, 2001). Baroody and Ginsburg (1990) cite similar examples where students were quite willing to accept answers that were clearly unreasonable.

Although formal mathematics teachers often rely on applied contexts as both a means to motivate students and as a source of problems, ultimately, the focus of mathematics is on abstract patterns. The context becomes part of the "...irrelevant detail that must be boiled away over the flame of abstraction in order to reveal the previously hidden crystal of pure structure" (Cobb, 1997, p. 76). Mathematical literacy however, depends on context. Whether the mathematical patterns and relationships have meaning depends on the situation in which they are found.

Gal (1997) reminds us that there are no "word problems" in life outside of the mathematics classroom. There are multiple contexts in adults' lives that require numbers, quantities, mathematical concepts, and algorithmic processes. These situations arise in such places as home, work, and during recreational activities, and include personal finance, informed citizenship, social action, further education, and active parenting. "High school mathematics organizes the principles in a very logical progression, but it does not teach the practice of mathematics. It is as if the designers of the curriculum were stuck in the notion that practice is the application of theory and will follow naturally when a student is well-grounded in theory" (Denning, 1997, p. 107).

When mathematics is taught in an abstract, lockstep manner, students may come to believe that only geniuses can understand mathematics, or that mathematics is just a bunch of facts and procedures that have to be memorized (Baroody & Ginsburg, 1990). Children naturally develop sophisticated numerical ideas and methods at home. School mathematics sends messages that often undermine children's confidences in these home methods. Because children have come to believe these messages, they abandon common sense and overlook their own practical knowledge. The failure of school mathematics to make a cognitive connection between children's own math-related knowledge and the school's version of math feeds a view held by many children that what they know does not count as mathematics (Butterworth, 1999; Baroody & Ginsburg, 1990; Resnick, 1995).

"For students to see the connections between different methods and concepts, and to be able to apply them in new situations, not only must their own ideas and techniques be acknowledged, ... but ideas and techniques learned in the classroom must be seen to be applicable in other situations" (Butterworth, 1999, p. 299). Failing to apply mathematics to a situation divorces the students' meaning of the numbers from the procedure – leading to mistakes that create unreasonable answers, without the student noticing.

Cobb (1997) suggests that we redirect public education at all levels by putting less emphasis on technical and purely mathematical skills and more emphasis on high-end mathematical reasoning. In order for students to be mathematically literate, Usiskin (2001) claims that we need to adopt the following practices:

- spend less time on small whole numbers
- give meanings of arithmetic operations that apply to numbers other than whole

numbers

- employ calculators and computers as a natural part of the curriculum
- combine measurement, probability, and statistics with arithmetic
- do not treat word problems that are not applications as if they were applications (Usiskin, 2001).

Steen (1989) states that, in a revitalized school mathematics program, topics such as the following should receive *increased* emphasis:

-Geometry and measurement
-Patterns and relations
-Probability and statistics
-Discrete mathematics
-Estimation and mental arithmetic
-Spatial reasoning
-Collecting data
-Observation and conjectures
-Real problems
-Graphical reasoning

And, in turn, several other topics should receive reduced significance:

-Fractions
-Graphing by hand
-2 column proofs
-Long division
-Paper and pencil algorithms

Steen does not recommend the deletion of these topics from school curriculum, but rather simply suggests a reconsideration of their importance. Although Alberta schools may not be ready for a reduced emphasis on fractions, those who redesigned the Alberta high school programs of studies have noted Steen's comments, and those of ones with similar beliefs. The *WCP*, on which the programs of studies are based, indicates four strands of mathematics to be studied from kindergarten to grade 12: number, patterns and relations, shape and space, and statistics and probability (Western Canadian Protocol,

1996).

Useful Mathematics

Frank (1988, 1990) found that certain student beliefs about mathematics can be counter-productive to achieving a solid understanding of mathematics. Students traditionally believe that mathematics is a collection of symbol manipulation rules, with some tricks thrown in for solving story problems (Resnick, 1988). The problem with this belief is that mathematics can be relevant, and, in essence, unless students find it relevant, they will be unable to see a reason for studying it. "The more connections that teachers can make with other subjects and with the personal lives of their students, the more likely that each student will find something of interest" (Noddings, 1994, p. 98). One of the reasons that students become disenchanted with mathematics is that they do not see its usefulness or relevance to their daily lives. Conversely, students may be more motivated to learn a content area if they perceive it as being useful (Steen and Forman, 1995). This finding something of interest to students may be the key to developing positive attitudes towards the learning and doing of mathematics in them.

An investigation into the combination of teaching approaches and the realistic constraints provided by contextual activities may provide insight into the factors that influence their mathematical knowledge outside of school.

> If students are unable to make use or sense of their school mathematics in such tasks within school, it seems unlikely that they will make use of this mathematics when similar tasks are encountered in the real world with an even greater complexity of mathematical and nonmathematical variables.

(Boaler, 1998, p. 53)

Furthermore, Boaler's (1998) research comparing two schools with differing mathematical teaching approaches found that the students exposed to a traditional, textbook approach were unable to make use of any school-learned methods in other situations "because they could not see any connection between what they had done in the classroom and the demands on their lives outside of the classroom" (p 58). In contrast, those students who were exposed to learning mathematics in an open, project-based environment "did not see a real difference between their school mathematics and the mathematics they needed outside of school" (p 58).

The NCTM would support Boaler's conclusions. They state, "school mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students" (NCTM, 2000, p. 15). Although mathematical topics can be considered important for a number of reasons, students who are struggling with mathematics, such as many of those in either M 33 or AM 30, generally only consider those topics in which they can see a societal connection as important. This notion of audience is paramount when discussing attitudes towards mathematics and the uses of mathematics.

A goal of mathematics education is to get students to realize and appreciate that mathematics is not only a powerful tool for understanding the world, but that it is also a powerful tool for discovering the world (Silver, Kilpatrick, & Schlesinger, 1990).

> Understanding and discovery are facilitated by a curriculum that has a strong component of exploration and investigation and is problem-based. With such a curriculum, students learn important mathematical concepts,

skills, and strategies as they deal with problems which may come from mathematics or physics, chemistry, biology, earth or space science, economics, political science, engineering, business, art, architecture, sports or music. (Greenes, 1995, p. 91)

Both M 33 and AM 30 were developed with the underlying principle that mathematics is a powerful tool for understanding and learning about the world. The M 33 diploma examination, which all students must write to complete the course, contains many questions that relate to industry, sports and recreation, and to environmental issues. However, because of the short period students have to write these questions, they needed to be designed as what Resnick (1988) calls story problems. "This representation… is a stripped down and highly schematic one that does not share the material and contextual cues of a real situation" (p. 56). The developers of the applied program recognized this and instead, attempted to use projects that engage a substantial amount of mathematical knowledge and ability to solve problems and investigate situations occurring outside of school. These contextualized problems help to integrate conceptual and problem-solving activities. To help support these teaching practices, test developers for the diploma examination in AM 30 went one step farther than the M 33 diploma examination. Not only were questions on the examination contextual in nature, but also a project was included as part of the large-scale assessment. The projects, intended to accompany each administration of the diploma examination, provide students with problems that can be solved by applying the mathematics they are currently studying (Alberta Learning, 2000a).

The problems designed by curriculum and resource developers are intended to be

moderately novel, and have some connection to students past experiences or to a possible area of interest. Ultimately, a problem should teach the students something about themselves, their world, and about the mathematics that they study. They are intended to be interesting to students because students are more likely to tackle and persevere with problems that tweak their interest (Baroody and Ginsburg, 1990). "The more connections that teachers can make with other subjects and with the personal lives of their students, the more likely that each student will find something of interest" (Noddings, 1994, p. 98).

Students' Attitudes Toward Mathematics

The Western Canadian Protocol (WCP) for mathematics places a high importance on students developing a positive attitudes toward mathematics. "It is important for students to develop a positive attitude toward mathematics so that they can become confident in their ability to undertake the problems of a changing world, thereby experiencing the power and usefulness of mathematics" (WCP, 1996, p. 9).

Many students disengage from school mathematics. This creates a serious problem, not only for their teachers, but also for a society that increasingly depends on a quantitatively literate citizenry. These students may find the subject as taught to be uninteresting and irrelevant or not of much use to them (NCTM, 2000). Or, perhaps it could be associated with repeated failure and negative interactions. Turner, J. C.; Cox, K. E.; Dicinto, M.; Meyer, D. K.; Logan, C. & Thomas, C. T. (1998) stated that negative feelings are common in mathematics classes where students frequently report being confused about the complexity and precision of the subject. These place students at great risk of experiencing negative affect (Yasutake & Bryan, 1995) which translates into poor

academic self-concept, low expectations of future academic performance, attribution of failure to low ability, and attribution of success to external factors (Licht, 1993).

Hart (1989) defined attitude as a "predisposition to respond in a favourable or unfavourable way with respect to a given object..." (p. 39), person, activity, etc. Haladyna, Shaughnessy and Shaughnessy (1983) found attitude to be stable rather than periodic, and generally, not an emotionally charged affective state. However, attitude does seem to involve emotional reactions, behaviours and beliefs (Hart, 1989).

In general, attitudes, beliefs, and emotions are the major descriptors of the affective domain in mathematics education (McLeod, 1992) whereas knowledge and thinking are considered descriptors of the content and process of the human mind (Brown and Borko, 1992). A number of researchers have investigated the relationship between the affective and the cognitive domains. Maher (1982) emphasized the importance of this relationship: "It is important to separate the cognitive from the affective domains in any activity.... The most important is that there is a cognitive component to every affective objective and an affective component to every cognitive objective" (pp. 30 – 31).

Neale (1969) defines attitude toward mathematics as an aggregated measure of "a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless" (p. 632). Ma and Kishor (1997) extended Neale's definition of attitudes towards mathematics to include "students' affective responses to the easy/difficult as well as the importance/unimportance of mathematics" (p. 27).

Krapp, Hidi, and Reinninger (1992); Prenzel (1988, as cited in Krapp, Hidi, & Reinninger, 1992); and Schiefel (1992) all conceptualize interest as a person-object

relation that is characterized by value commitment and positive emotional valances. Koller, Baumert, & Schabel (2001) suggest that interest is an important determinant of academic choices. Interests influence academic achievement and learning in school.

As schooling progresses, students' attitudes toward mathematics seem to become less positive (Ma & Kishor, 1997; Utsumi & Mendes, 2000; Koller, Baumert, & Schabel, 2001). Utsumi and Mendez surmise that this may be associated with a decrease in understanding of the subject or content. In a study of 602 students in Germany, Koller, Baumert, and Schnabel (2001) found that interest in mathematics decreased from grade seven to grade twelve. "One attempt to explain this trend assumes a mismatch between the curriculum and the general interests of adolescents" (p. 465). Schoenfeld (1988) and Travers (1978) claim that math and science instruction typically ignores the everyday life experiences of students.

There appears to be a correlation between achievement in mathematics and attitudes towards it. Students who enjoy mathematics and perceive it to be useful, generally do better in mathematics, but the converse does not necessarily hold true (Mullis, Martin, Ganzalez, Gregory, Garden, O'Connor et al, 2000; Ma & Kishor, 1997; Fennema & Sherman, 1978; SAIP, 2003). In general, it is impossible to tell just from the correlations whether those who like mathematics become good at it, or those who are good at mathematics come to like it.

In the literature, motivation has been increasingly treated as a function of cognitive decision-making. Weiner (1984), Covington (1992), Ames (1992), Blumenfeld (1992), and Elliot and Dweck (1988) all assume that individuals only put forth effort when they believe that effort will result in fulfillment of their goals. In contrast,

individuals who attribute their performance to effort rather than ability or who have positive self-perceptions of performance generally try harder and persist longer at difficult or challenging tasks (Dweck, 1986; Lich & Dweck, 1984). These students, according to Sternberg (1985), are more likely to succeed than those with negative perceptions.

Student alienation in the middle years is reported to derive from a curriculum that does not recognize the personal and social context of young adolescents (Cumming, 1996; Eyres, 1997). Children (and adults) filter and interpret new information in terms of their existing knowledge.

> [Students] cannot assimilate new information that is completely unfamiliar. Quite naturally, they quickly lose interest in the incomprehensible information and tune it out. Somewhat unfamiliar information can be related to existing knowledge and assimilated. Children are naturally interested in *moderately novel* information. Assimilation and interest then, go hand in hand. Like adults, young school children of do not make the effort to assimilate new information unless it makes some sense and hence is important to them. When a task piques their curiosity, children will spend considerable time and effort working at and reflection upon it. (Baroody & Ginsburg, 1990, p. 56)

Research in mathematics education suggests that more than content knowledge is required to be successful in mathematics, and a learner's ability to master mathematical content is shaped by the learner's attitude toward the content. Attitudes play a critical role in establishing and maintaining a continued interest in mathematics (Eccles, Adler, &

Meece, 1984; Pedro, Wolleat, Fennema & Becker, 1981; Tobias, 1987).

The research reviewed in this chapter speaks to the need to listen to the voice of the student when designing, and teaching mathematics curricula. The literature suggests the following three main ideas:

- Students learn and co-evolve within their environment. Mathematics curricula must be designed, and teachers must create lessons under this umbrella. We as teachers must release our need to control students' learning and allow them to accept more responsibility, and we must release our fears that the content will not be covered.
- The need for mathematically literate citizens is paramount, and in order to develop such citizens, school mathematics must relate to the world outside of school when possible. Mathematics must be taught in context in order to develop students' confidence in applying appropriate mathematics in numerous situations.
- To build and maintain students' interest in learning and doing mathematics the curriculum must relate to some of the interests of adolescents. Students must consider the mathematics that they learn to be interesting, relevant, and something in which they feel confident.

From these three ideas that have emerged from the related literature, my areas of interest have developed. The two programs in which the participants of this study are enrolled both use different approaches to a similar end. Both courses were developed with the purpose of allowing students to construct their understandings of mathematics, of developing mathematically literate citizens, and of building students' positive attitudes towards mathematics. I became interested in understanding the experiences of the

students enrolled in these programs, and how the different approaches may affect the students' attitudes and beliefs about mathematics.

CHAPTER III: METHODOLOGY

When I first began to envision a study regarding students' attitudes towards mathematics, I was reminded of the question often asked by students, "When will I need this?" – a question that seems to plague most high school mathematics teachers. This all too common question became the foundation for considering the method of collecting information about students' beliefs and attitudes towards mathematics.

I believe that stories of students' mathematical experiences, their beliefs, and their subsequent attitudes towards mathematics could be told through their own words. My job, as the researcher then, became one of asking the right questions to elicit the stories, and to listen carefully enough to hear them. I decided that the best method through which to achieve this goal, and tell these stories, was to use a case-study methodology. Case study research is used rather extensively in educational research. "[E]ducation has recognized the advantage of using a case study approach for better understanding the process or dynamics of certain aspects of practice" (Merriam, 1988, p. 204).

Research Design

The research design I chose for this study is a descriptive case study. The data, collected from two different cases, describes the perceived significance that school mathematics plays in the lives of the students involved in the study. I collected qualitative and quantitative data through questionnaires, interviews, and observations that helped to describe students' attitudes toward mathematics; some of their experiences that may have lead to those attitudes; and the aspects of mathematics class that students said they

enjoyed or did not enjoy. This was done in an attempt to not only report the experiences of students, but also to interpret and explain, as best as I could, the significance of some of these experiences. Students were asked to indicate their views and preferences using a 5-choice Likert scale (see Appendix), and describe some of their experiences in greater detail. This led to some of the data being quantitative while most was qualitative in nature.

Research Methodology

Qualitative research

Stake (1995) views the differences between qualitative and quantitative research to lie not so much within the nature of the data collected, but rather in what the researcher searches for. The qualitative researcher searches for happenings and understanding while the quantitative researcher searches for causes. Qualitative research methods seek to understand human experience while some quantitative methods seek to find cause-and-effect relationships. I was aware, well before the research started, that the issues surrounding beliefs and attitudes towards mathematics were too numerous and intertwined to seek any cause and effect relationship. Rather, mine was a quest to understand these students' experiences, not to determine their causality. While many discrepancies in student-responses may be attributed to the differences between the course content and teaching styles, no definite relationship can be determined by this research.

Through this research I will describe the situations and occurrences regarding the two different approaches to mathematics education, and look for implications that will

inform future mathematics curricula and teaching approaches. Hence, this research considers two cases. My question is not why there may or may not be differences between the two groups, but rather I am interested in exploring the two situations. As a researcher, I attempt to create a "complex, holistic" (Creswell, 1998) picture of the subjects, their motivations, and their beliefs. Most similarities or differences that emerge from the research encompass too many variables to be studied quantitatively. Although I will report some quantitative, descriptive data, a qualitative research method is more suitable.

Case study research

I have chosen to utilize a descriptive case study as my methodology because I believe it to provide the best opportunities for describing the classrooms, teachers, students, and their beliefs. Although the definition of case study differs slightly among the experts – Stake (1995) presents case as an object of study, or a bounded system, while Merriam (1988) describes case study as an approach or methodology – I will use both of these definitions. This study is bounded by the time frame of one semester, by the physical space of the schools, and by the approaches and curricula of the two courses. My approach to the research also centres on an attempt to build a scene that accurately depicts the outlooks and beliefs of grade 12 mathematics students towards mathematics. My research centres on two cases – a M 33 class and an AM 30 class, and therefore could be described as a collective case study.

The research is inductive rather than deductive in nature (Merriam, 1988). The variables that may have led to the results observed in these two classrooms were obviously too numerous to identify before embarking on the research. Plenty of variables

were only identified through interactions with the participants of the study, and many were probably never identified, due to the limitations of time and scope of the research.

Reliability and validity

In this research, I have taken a number of steps to ensure validity (both internal and external) and reliability (Merriam, 1988). Internal validity – ensuring that I captured and portrayed the case as it appeared to the participants was achieved through triangulation, member checks, observations over a period of time, and checking findings against the teachers' perspectives.

Information gathering occurred via a number of sources of information, to allow for triangulation (Creswell, 1998; Merriam, 1988; Stake, 1995). I administered questionnaires to students, observed classes, and conducted interviews with selected students. I also conducted ongoing informal interviews with teachers as a fourth method of strengthening my interpretations of the information that students had provided me. My intent was to use multiple sources of data, to gain multiple perspectives, and to reduce the risk of misinterpreting any particular action or discussion point.

These actions also speak to the reliability of the study. The multiple methods of collection and multiple representations of data build dependability and consistency in my interpretations – that is, given the collected data, the reader should be confident in my interpretations. The reliability of this research then, is not considered the same as reliability of a study done in traditional experimental research. Rather than concentrating on whether the study can be replicated, I am more concerned with whether the results are dependable (Merriam, 1988; Walker, 1980).

Of course, the purpose of research and presentation of research is to contribute to

the mathematics education community, provide an opportunity to reflect on mathematics education, and provide the reader something from which they can learn. The question may arise – How can a reader learn from a single case? My intent is to have the reader learn through *naturalistic generalizations* (Stake & Trumbull, 1982), that is, conclusions at which the reader should arrive because the experiences are presented in a fashion as to provide a vicarious experience. I provide heuristic descriptions of students, classrooms, schools and programs being studied. I have tried, wherever possible, to include *thick descriptions* of the environment, the topic being studied, and the actions of the students (Merriam, 1988; Stake, 1995; Creswell, 1998).

Context of the Study

This research involved working with two schools, both neighbouring a large urban area, and both serving similar communities in terms of socio-economics and political orientation. I studied the responses and actions of students in these two schools – M 33 students in one school and AM 30 students in the other. My intention was to investigate the attitudes expressed by students who were enrolled in M 33 and those of students enrolled in AM 30 using questionnaires, interviews, and discussions with their teachers. The questionnaire was administered to all students in these classes who agreed to participate. Selected students, and the teachers from both classes were interviewed. There were 21 M 33 students, from a class of 31, and 14 AM 30 students, from a class of 18, who agreed to participate in this study.

The jurisdiction

The jurisdiction to which these schools belong has urban schools, rural schools,

small schools and large schools. The jurisdiction includes one city of approximately 15 000 people, in which one of the schools involved in this research study is located, and one large town of approximately 10 000 people, in which the other school involved in the study is located. Although there are 21 schools in the jurisdiction, serving approximately 9 300 students, the two schools involved in this study are the only two senior high schools.

The schools

The schools in which I conducted this research are large schools (population >1 000 students) and both have strong reputations in their respective communities. One school has been in existence for 52 years, and the other for 25 years.

Each school has a relatively large mathematics department, with more than eight full or part-time mathematics teachers. All of these teachers have been teaching for more than five years, with the average teaching career among the departments being approximately 15 years. Obviously then, both schools have well established, experienced mathematics departments. Teachers in both departments meet regularly through a division-sponsored instructional support program, to discuss issues related to curriculum and instruction. Because of this, mathematics teachers in both schools share similar experiences, and have had common opportunities to reflect on mathematics education.

The teachers

The two teachers whose students participated in the study are both experienced high school mathematics teachers, each with more than ten years experience. For both of them, those ten years have been within the same school jurisdiction. These teachers are respected by their students, by colleagues, and by members of the community. In

addition to being experienced mathematics teachers in general, both teachers have taught the courses in which I conducted interviews at least twice in the past. This is important to the research because it avoids the possibility of either teacher feeling uncomfortable with the content of the course, and simultaneously having to deal with a researcher within their classroom; or, either teacher subconsciously exhibiting more enthusiasm for the program than they otherwise would because of the newness of it.

The students

The students included in the study were grade 12 students aged 16 to 18 years old. These students are enrolled in a mathematics class that will gain them only limited access to many programs at university. The post-secondary institutions that they can gain access to through successful completion of M 33 or AM 30 include some programs at colleges or at the Northern (or Southern) Alberta Institute of Technology. My experience is that the students enrolled in these programs, generally speaking, are not those who thoroughly enjoy mathematics (those students would most likely be enrolled in PM 30 or Mathematics 31), and they are not enrolled in this class because they require it to get into a university program.

In my experience, students in these mathematics programs are more likely to either discount the importance of mathematics, both to themselves and to society in general, or to find importance in mathematics outside of the world of school. These students do not express that they experience pressure to enroll in university or another post-secondary institution. However, the pressure may still be there. Many of these students may be enrolled in the grade 12 mathematics classes that they are because they have been convinced (either by their parents, school counselors, or teachers) that they

need a grade 12 mathematics course. They may be taking mathematics, not necessarily because they enjoy it, but because they have never questioned their need for it.

Data Collection

Observational tools

I observed students during three mathematics classes, each for a total of four hours. Classes I observed were selected in conjunction with the teachers. They were classes that the teacher believed best represented her or his teaching style and the intended curriculum, or in which they had chosen a rather novel teaching approach.

My observational tactics were unobtrusive, as I sat at the back of each classroom while observing, and included keeping ongoing field notes when observing classes. As such, I attempted to play the role of a complete observer. I did not participate in class discussions or with the students unless specifically asked a question. However, all of the students who participated in the study knew that I was a high school mathematics teacher, and, as such, felt comfortable approaching me for assistance when they were working on class assignments and the teacher was otherwise occupied.

My notes concentrated on the topic that the teacher was teaching, and the interactions and questions that students put forth. These observations, and the subsequent field notes helped to inform my selection of students to interview, and provided situations to which I could refer during my interviews with those students.

I worked under the assumption that through students' actions (or lack of actions) I could identify students who seemed interested in the topic being taught and those who were clearly not interested in the topic. However, one aspect that I was interested in observing was whether or not students' actions in class did or did not parallel their

responses in the questionnaire. That is, I was curious to see if any students appeared interested in class, but gave answers to their questionnaire that did not indicate an interest, or vice-versa.

My presence in either classroom did not appear to affect the manner in which the students interacted with their teachers, or with each other. Both teachers observed used mainly teacher-centred practices, and most students very quickly ignored my presence. On more than one occasion, I checked with each teacher, to see if the behaviour on any particular day that I observed was typical. On each occasion the teacher indicated that it was indeed typical. From this I concluded that my presence had very little effect on the students' behaviour.

Questionnaire

Students were asked to complete a questionnaire regarding their experiences and attitudes towards mathematics and the mathematics course in which they were currently enrolled (Appendix). This questionnaire was developed with the intent of extracting perceptions about mathematics from the participants. It included both closed-ended questions, asking students to rate their preferences using a Likert scale, and open-ended, short-answer questions asking them to relate experiences and to describe events. Particular sections of the questionnaire related to sub-questions of my research question. There were questions relating to students' enjoyment of mathematics class – a rating of their level of enjoyment, types of activities they enjoyed or did not enjoy (questions 1 – 5). There were questions that related to the amount of work that students perceived that they put forth in mathematics class (questions 6 – 8), and perceived usefulness and relevance of mathematics – use of mathematics in other classes, or outside of school

(questions 9 – 12). Still others related to the level of success the student had achieved, and to what they attributed that success (questions 13 – 15).

The questionnaire was developed over time through my reading of other researchers findings in their studies of students' attitudes toward mathematics and my reflections on my teaching experiences with high school mathematics students. There appears to be a correlation between achievement in mathematics and attitudes towards it. Students who enjoy mathematics and perceive it to be useful generally do better in mathematics, but the converse does not necessarily hold true (Mullis, Martin, Ganzalez, Gregory, Garden, O'Connor et al, 2000; Ma & Kishor, 1997; Fennema & Sherman, 1978). Funkhouser (1993) found that the introduction of problem-solving software into a mathematics class coincided with a dramatic increase in the number of students who indicated that they would like to take another mathematics class. Readings of these studies led me to ask questions about whether or not students enjoyed mathematics class, what types of mathematical activities they did or did not enjoy, and whether they were interested in taking any more mathematics classes.

In a 1997 study, Higgins found that students did not enjoy doing problems that they found to be irrelevant with no application to the world outside of school. Higgins' results led me to consider asking students the questions that related to their beliefs about the relevance of the topics they were studying, whether or not they could identify any uses of the mathematics they studied in any of their other classes, and whether or not they considered mathematics to be important to themselves and to society in general.

Dweck (1986) found that individuals who attributed their performance to effort rather than ability or who had positive self-perceptions of performance generally tried

harder and persisted longer at challenging tasks. Attribution theory (Weiner, 1984), self-worth theory (Covington, 1992) and goal-orientation theory (Ames, 1992; Elliot & Dweck, 1988) all assume that individuals only put forth effort when they perceive that effort will result in fulfillment of their personal goals. Kloosterman (1998) found that students, when deciding how hard to work in mathematics class, considered perceived usefulness of the mathematics that they were studying. The readings of these researchers pushed me to ask questions relating to students' perceptions of how hard they worked in mathematics class and how much mathematical ability they thought they possessed.

Interviews

Once my classroom visits were complete, and students had completed the questionnaire, I reviewed my observational notes and student-responses to the questionnaire and from that analysis, selected five M 33 students and six AM 30 students to interview. The purpose of the audio taped interview was to help me clarify the students' responses and to the questionnaire, to ask them about specific observations I had made involving them, and to gain further insight into their statements. Students were selected for interviews on the basis of unique or interesting actions in class, and responses to their questionnaire. I attempted to select both students whose questionnaire responses did not seem to match their actions in class, and students whose questionnaire responses did seem to match their classroom actions. I also sought input from the teachers to inform my choices of students to interview. These students were subjected to semi-structured interviews (Frankel & Wallen, 2000). All students were asked about the other classes they had in that semester and what they thought about mathematics class in comparison to those other classes. Each were then asked questions specific to their

actions in class or to responses to their questionnaire. For example, if a student indicated that they worked hard in mathematics class on their questionnaire, I asked them what they thought working hard meant, and whether they thought their teachers would be in agreement with that statement.

Data Analysis

Upon completion of the data gathering, my task was to compile the collected data and discern themes that arose. I looked for differences and similarities in actions and responses between students who enjoy mathematics and those who do not enjoy mathematics, between students who believe mathematics to be important and those who do not, and between students enrolled in AM 30 and those enrolled in M 33, as well as other differences that arose from responses to the questionnaire, interviews and observations.

Some of the questions that students were asked to respond to used a Likert scale to elicit responses. Thus, these descriptive statistics could be best described and summarized through the use of bar graphs and trend analyses to show relationships. My intent in doing this was not to reduce this information to a display of numbers and charts, but rather to present it in a descriptive form, and as richly as possible.

The classification of the themes stemmed from the research sub-questions that I posed. I entered transcripts from the interviews and answers from questionnaires into a database so that I could group student responses by the categories listed above. From these groups, I was better able to conceptualize patterns and characteristics (Merriam, 1988). It also allowed me to seek meaning from single instances instead of only looking

for generalizations (Stake, 1998). I also used my field notes from my classroom observations to compare responses to questions on the questionnaire and the interview with the manner in which the students acted in class. When comparing students' responses to the questions involving a Likert scale, I used a graphing calculator to view a scatterplot of their responses, and calculate the correlation coefficient of a linear regression. The graphing calculator calculates a Pearson product-moment correlation coefficient (r).

During the observations, my objective was to watch, listen, and consider the situations, as closely as I could in order to gain the maximum amount of insight from the students and their teachers. In the following chapters I have attempted to clearly present these situations to the reader in order to show patterns, characteristics, and individual occurrences. The purpose is to tell the stories of the students through their own actions – making them actors in the descriptions rather than background props.

Delimitations

Since the focus was on collecting information about the attitudes of students in these classes toward mathematics, I did not conduct formal interviews with the teachers. I discussed, informally, the perceptions of the teacher in relation to students' responses to the questionnaire, and any other comments they wished to make regarding the class or the students.

I observed classes of the teachers' choosing, but limited these observations to only three class periods per group, for the purpose of correlating students' responses to the questionnaire to their actions in class. I chose not to obtain information about the specific results of student performance in the classes, choosing instead to ask those

students who I interviewed to describe how they were doing in math class compared to their other classes.

Limitations

My observations were limited to two classes in one semester. The M 33 course was not being offered past June 2002, and the AM 30 course would be mandatory after that. There were other sections of both courses being offered, but one of the other teachers was teaching AM 30 for the first time. I chose to avoid this situation for the reasons outlined earlier. Also, working with additional teachers who were teaching M 33 while avoiding those who were teaching AM 30 for the first time would have resulted in a significant difference in the number of students from each class involved in the study. For this reason, I was unable to obtain results from more than one class each.

My findings are also limited by the number of students in each class that agreed to participate in the study. In each class, over one half of the students chose to participate – 21 of the 31 students enrolled in M 33, and 14 of the 18 students enrolled in AM 30 agreed to be observed, interview, and/or agreed to respond to the questionnaire.

One final limitation is the possible discrepancy between students' perceptions, my observations, and the observations of the teacher. Although some students may appear to work hard in math class, or possess a great deal of mathematical ability, my observations may lead me to a different conclusion. I hope to minimize this discrepancy by comparing my observations with the students' responses to the questionnaire, and with their answers to the interview questions.

In the next two chapters, I offer discussions of each case. Chapter four discusses my observations and the responses from the M 33 students, while chapter five discusses

my observations and the responses from the AM 30 students.

CHAPTER IV: LISTENING TO MATHEMATICS 33 STUDENTS

The following case describes the experiences, perceptions, and attitudes of students enrolled in M 33, the course that emphasizes an inductive development of topics in geometry, algebra, trigonometry, and statistics. They are based on classroom observations, responses to the questionnaire, and interviews with selected students and with the teacher.

Mrs. Stewart and her Classroom

Mrs. Stewart's mathematics classroom is a standard sized, portable classroom that has been permanently attached to the school. There are posters on the walls, most of which relate to mathematics, study habits, and post-secondary institutions. There is one small window in the room that looks out on to the grass between this high school and a nearby K-9 school. In relation to many of the classrooms in this school, this room is a desired room, simply because of that window. Mrs. Stewart's desk sits at the back of the room and there is an overhead projector and screen at the front left corner. Students' desks are arranged in pairs around the room, and students are encouraged to discuss questions with those students sitting next to them. By and large; this is a typical high school mathematics classroom.

Mrs. Stewart's teaching style is somewhat traditional. A typical class period in her M 33 class will involve an exploration or discussion of the topics, some worked examples in which students participate, and finally some class time for students to practice. During this time, Mrs. Stewart walks around the room, answering questions or trying to explain concepts again to students who are having difficulty. She encourages some group work

and designs some activities that centre more on the discussion of mathematics rather than direct teaching. I did notice however, that Mrs. Stewart's teaching methods generally help students to develop procedural understandings and place less emphasis on developing relational understanding (Skemp, 1987). I was unable to determine whether this is her general teaching style, or if it was unique to this class.

What the Students Said

Enjoyment and interest in mathematics

Students were asked to rate their level of enjoyment of mathematics on a scale of 1 to 5, with 5 being strongly enjoy. A chart indicating the distribution of their responses follows:

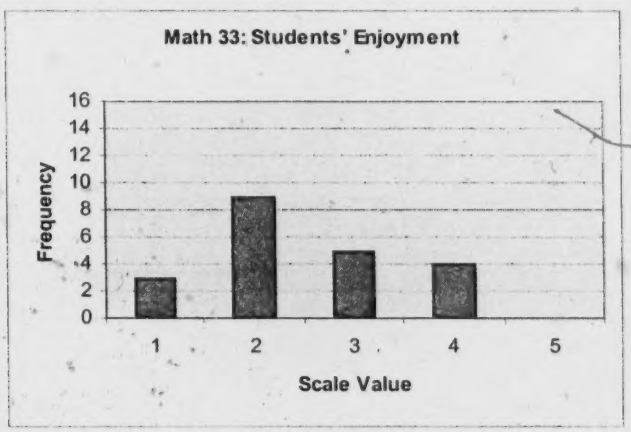


Figure 2: Math 33 Students' Enjoyment Rating

Of 21 participants from this M 33 class, only 19% of them claimed to enjoy mathematics, as indicated by rating their enjoyment at 4 or 5 on the scale. Of these students, their subsequent comments seemed to indicate to me, more of a lack of hatred of mathematics rather than a true enjoyment of the subject. One student, who rated his

level of enjoyment as 4 out of 5 wrote on the questionnaire

> *I can't say that I have ever not enjoyed mathematics. It's not my favourite class, but I don't mind it.*

Those students who rated their enjoyment of mathematics at a 4 or 5 were asked a subsequent question: Do you remember ever not enjoying mathematics? Describe the activities or events that occurred during a math class that you did not enjoy. Many students wrote that they remembered classes in which they wrote a great deal of notes.

> *During a whole class in grade 9, all we did was take notes from the board then answer repetitious questions. We didn't have any discussions [or] projects ... and I got very bored and would just zone out and wouldn't learn as well.*

Other students expressed a dislike for math when they didn't understand a topic and found the experience frustrating.

> *I remember not enjoying a math class when I don't understand a topic and therefore I struggle through the unit and hate coming to class. In Math10 Pure I didn't understand a unit and didn't come to class to struggle and therefore my mark dropped.*

Other students made similar comments.

> *If I don't know what I'm doing I get frustrated.*

Although the number of students who indicated that they strongly enjoyed mathematics was quite small, there did seem to be a common theme that emerged from their responses when asked if they had ever not enjoyed mathematics class – one of boredom, and of finding mathematical topics difficult to comprehend. No students mentioned rating their enjoyment level based on their relationship with any particular teacher.

Slightly more than one half of these M 33 students (12 out of 21) indicated their level of enjoyment of mathematics to be 1 or 2 out of 5. Of these 12 students, six could

recall a time when they enjoyed mathematics, and all but one of these six students referred to elementary school. For example, Travis, who indicated his level of enjoyment as 2, wrote

> *Anything before grade 7. Why? Algebra, I hate it. Math used to be my strongest class; as soon as algebra came into the picture it dropped to my weakest class.*

Cara, who indicated her level of enjoyment of mathematics as 1, remembered enjoying math class when she was in grade 4:

> *In grade 4, my teacher was very hands-on and all the math classes involved blocks, pictures, going outside, etc..*

Robyn (1) recalled liking math in grade 6, where she was involved with Problem-Solving Fridays. As she wrote: *I was good at it.*

Other students remembered enjoying mathematics in the elementary grades because they could play games and get rewarded with candies or treats, but none of these students mentioned ever enjoying mathematics class because they found the topic interesting or the nature of mathematics and patterning to be intriguing. All of their recollections of enjoyment centred on games, competitions, and treats.

One particular student, Jason, captured my interest because of the apparent discrepancy in what I observed in class, what he wrote on his questionnaire, and his comments during the interview. During a lesson on trigonometry, I tried to observe Jason quite closely. The following are excerpts from my field notes:

Box Plot Lesson: Jason sits directly in front of me. He is slouched down in his desk and is looking around the room or down at the floor. Mrs. Stewart is discussing the uses of box and whisker plots. She has in her hand a box of pasta that contains regular and spinach pasta. She is using them to show the probability that a randomly selected group of 20 pieces of pasta will contain x numbers of green (spinach) pasta. Students are asking questions and each counting out 20 pieces of pasta. Jason quietly counts out his pasta, and goes to write his results on the board, goes back and sits down, and begins to write something down. He rarely looks up again during the lesson.

On another day, I observed a lesson dealing with trigonometry and angles in standard position. Jason's participation level seems to be even lower than the previous lesson:

Trigonometry Lesson: Again, Jason sits in front of me. Mrs. Stewart has posed a problem for students to solve, and is offering candy to the first person with a correct response. Jason does not appear to be attempting the problem. He has not picked up his pencil. I wonder whether he knows how to do the problem, or is he just not interested in doing the work. Mrs. Stewart poses another problem, and offers the same reward. Jason punches some numbers into his calculator, writes something down on his paper, and puts down his pencil again. He does not offer an answer (even though it appears that he has one), nor does he attempt to confirm his answer with those students around him, as most other students are doing.

In both of these lessons, Jason does not appear to be interested in the topics at all. He appears to put forth minimal physical participation into the lessons by not writing down any detail related to the examples, and his verbal participation is virtually non-existent. However, Jason's questionnaire revealed an interesting side to his quest for knowledge.

Jason wrote in his questionnaire how mathematics was a universal language that everyone uses. "*Nearly everyone in society can be linked to math.*" He claimed that mathematics was important to him because "*it is one of the fundamentals of success....*

Knowledge is power, and yet ignorance is bliss. I quest for knowledge like a miser quests for riches." He claims to participate rarely because he hates being put on the spot.

During his interview, Jason said he did not know what aspects of this mathematics class would be applicable to his future, but he guessed that quadratics and algebra might be. When asked why, he responded that he expected he would need algebra for some of the classes he planned to take at the technical school he would be attending the following term. However, he could not think of any classes in high school in which he had used any of the mathematics that he had studied. Jason could not think of any units or topics that he had studied in M 33 that might be helpful in his everyday life outside of school. As with most of the M 33 students I interviewed, Jason could not think of any examples of mathematics that would relate directly to his life outside of school. As I interpreted Jason's actions and responses, I think that Jason believes high school mathematics to be a subject that he has no choice but to study. It has no real place in the lives of people outside of school – even those in the seemingly mathematically intense field of computer technology.

I later learned that Jason had an honours average in Math 33. I wonder now if his minimal participation revealed an ability to visualize the problem without writing it down, and then being able to solve the problem with a few keystrokes on his calculator. It should be noted that Jason's actions were by no means unique. There were plenty of students who put forth minimal effort and participation in class. Jason however, was the only student whose comments on the questionnaire and interview seemed to contradict his actions in class.

How hard do I work? How much ability do I have?

Students' level of enjoyment of mathematics was compared with their perceptions of how hard they worked in math class and how much mathematical ability they had, and to which of these two aspects they could attribute their success. Along with rating their level of enjoyment on a scale from 1 to 5, students rated how hard they worked, and how much mathematical ability they had on the same scale. Those students who were interviewed were asked to clarify their view on the amount of work they did in math class – what they did in class that showed that they worked hard, or if they did not feel that they worked very hard, what they would need to do differently if this were to change. A graphical representation of the students' responses to these questions follows in figure 3 and figure 4.

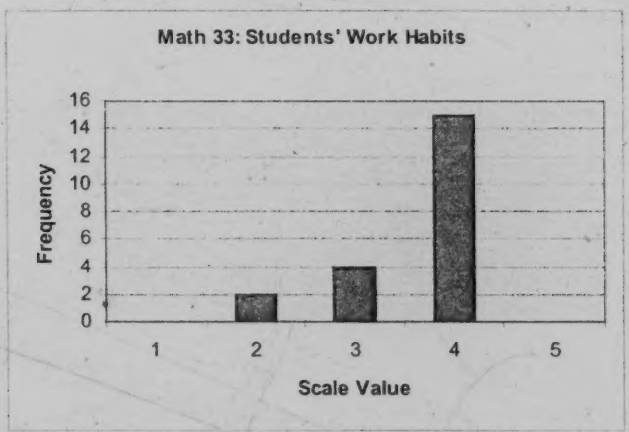


Figure 3: Math 33 Students' Work Habits

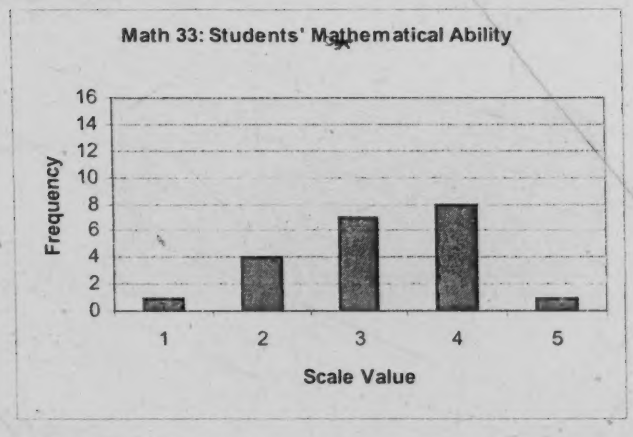


Figure 4: Math 33 Students' Perceived Ability

When relating students' level of enjoyment to how hard they worked, I used a graphing calculator to calculate the Pearson product-moment correlation coefficient (r). The mean score for enjoyment of mathematics was 2.5 out of 5, (SD = 1.0). The mean score for how hard they perceived they worked was 3.7 out of 5, (SD = 0.6). I did find a weak, positive correlation (r = 0.4) between the participants' level of enjoyment and their perception of how hard they worked, but this correlation is not acceptable at the 0.05 level of significance.

The higher a student indicated their enjoyment of mathematics, the harder they also perceived they worked in class. A perception of how hard they worked in math class however, being such a subjective notion, was different from one person to the next. During interviews, students were asked if they thought they did work hard in class, what that meant to them – what kinds of things did they do that showed that they worked hard. Some students (like Teresa) claimed that they worked hard in class, and that meant that they did their homework all of the time, paid attention in class, and didn't talk too much in class. Other students (like Jason) thought that he worked hard in class by getting as

much homework done in class as possible, so that he would not have to take any home. Jason said that he rarely did mathematics outside of class time. I used the same method of regression analysis to compare the relationship between how much a student indicated that they enjoyed math class and how much mathematical ability they believed they had. The mean score for how much mathematical ability students thought they had was 3.2 out of 5, (SD = 1.0). This relationship was again positive, and significantly stronger (r = 0.73) than the relationship between enjoyment and how hard they worked. This relationship can be accepted as linear at the 0.01 level of significance ($t = 4.69$, $N = 21$, $t_{19, 0.005} = 2.36$). Although this relationship had a positive, and relatively strong correlation, the students generally did not think that they had much mathematical ability.

Regardless of their level of enjoyment, all students were asked to generalize about the types of mathematical activities that they enjoy the most, and the ones they enjoy the least, the activities they found interesting and those they found uninteresting. These students seemed to associate enjoyment and interest in mathematics and mathematical activities with the level of understanding they could obtain. Six students specifically mentioned enjoying activities or lessons that were easy to understand and laid out clearly. That is,

> *the ones that use common sense. I don't like using formulas that I don't understand. For me, I like to know every step and how it works. So that there turns me off most algebraic math because it uses complex formulas that to me don't have much meaning. In other words, I like the easy stuff.*

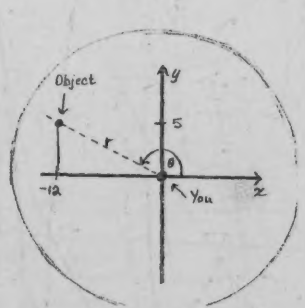
Some participants indicated that they enjoyed lessons in which they could become actively involved. As an example, eight students mentioned one particular M 33 class in which their teacher used masking tape to draw a coordinate plane on the floor and had students form functions using rope, to explore domain and range. As one student put it, "*I*

remember this because it was definitely different than our average math class where you just take notes and do assignments."

Six of the students indicated that they found activities involving trigonometry enjoyable, or that they were interested in trigonometry. None of these students gave particular reasons as to why they enjoyed trigonometry lessons more than others, but some did provide reasons for their interest in the topic. One student wrote *"I found trigonometry interesting because I could relate to finding the height of an object."* Another student indicated that trigonometry was something to which she could relate. She wrote *"It involved using common occurrences in math and I just felt it was easier to understand."*

I was able to observe one of the M 33 lessons in the trigonometry unit. Mrs. Stewart started the lesson by drawing a diagram of an angle in standard position on the board. The terminal arm of the angle was in quadrant II and the students were given the following scenario:

Mrs. Stewart: *"You are viewing a radar screen. The guy who usually looks after the radar screen is sick today. Your supervisor knows that you have studied trigonometry in math and asks you to fill in. This is a diagram of what you see on your radar screen. You need to determine the distance the object is away from you, and the measure of angle theta, so that you can inform your supervisor."*



Mrs. Stewart continued on throughout the entire lesson using this example of a radar screen as the basis, changing the location of the object, and working with different quadrants of the coordinate plane. This tie to a world outside of school may be an example of the reason that so many of these students found trigonometry to be applicable and relevant. There were two students however, that indicated how much they disliked trigonometry because it was a lot of work, and they found it confusing.

Why am I in this class?

Students were asked why they were registered in M 33 instead of any other mathematics class offered at the high school such as PM 30, or Mathematics 24. Every one of the students who completed the questionnaire indicated that they were taking M 33 because they either had attempted courses in the PM 10, 20, or 30 stream and found them to be too difficult, or they learned that M 33 was the minimum requirement for entrance into their chosen field of study. I did not find it surprising that the reason that these students chose to enroll in M 33 was not because of a difference in topics or approach to

topics than PM 30. M 33 contains much the same content as the PM 10 and 20 courses, but spread over three courses (Mathematics 13, 23, 33) instead of only two.

Will I take another math course?

To further check students' interest in and enjoyment of mathematics, they were asked if, given the choice, they would ever take another math class. Of the 21 participants, 3 (15%) indicated that they would take another class in mathematics if they had the opportunity. All other respondents replied with either an emphatic no, or a definite preference for not taking any more mathematics classes unless they were required. Reasons given by these students generally indicated that they found mathematics to be difficult for them to learn and they were too frustrated to continue past a required point, or that they just disliked studying mathematics and had no desire to do it any longer than necessary. As an example, Jackie wrote, *"I would not like to take any more courses because I find that the math curriculum sucks. I think they should make it more [applicable] to real life."* and Sam wrote, *"I wouldn't like to take any other math classes because [of] a lack of ability and knowledge. I may need to take Math 30 to fulfill requirements at university."*

Participation, Difficulty, and the World Outside of School

There were three themes that seemed to emerge from the students' responses with respect to enjoyment of, and interest in mathematics. These students were concerned with their participation in class, the difficulty level of the mathematics, and how their studies related to the world outside of school.

Active participation

The participants had a need to be active participants in the lesson. These students indicated that they were easily bored in math class. One student indicated that she did not enjoy "*sitting in class listening to the teacher talk the whole time*." Seven students made similar comments to Cara, who indicated that she did not enjoy "*boring, long, drawn-out talks with lots of note-taking*." Chelsea indicated that she disliked doing questions out of the textbook.

Difficulty of the material

There did not appear to be any consensus on the difficulty level of the course. Some students mentioned that they found M 33 (or at least topics within it) to be too difficult, while others claimed that mathematics was repetitious and boring because it was too easy. For example, Kathy wrote that she found math uninteresting when she was given math pages to do, even though she found what was learned in class to be easy. "*It's boring doing the same type of questions over and over when you already know how to do most of them*." However, two students indicated that they especially did not enjoy the unit on rational expressions because they found it to be too difficult.

The world outside of school

The last theme related to a need for school mathematics to relate to the world outside of school. One-third of the students claimed that they enjoyed lessons relating to everyday use. For example, Sarah mentioned that she enjoyed hands-on projects or activities that illustrate or put to use the topics they study, such as building models and putting their knowledge to practical use. Janine also claimed to enjoy activities that could be related to her life. As examples, she gave working with interest rates and cost of

living, although she wrote that she had difficulty understanding the various formulas and how to rearrange them. This indicates, I believe, that students, although not generally interested in activities that they find overly challenging, can still find uses for topics that relate to their lives, regardless of how difficult they may find them.

Importance and Relevance of Mathematics

All of the participants in the study were asked about the importance of mathematics, both to themselves and to society. As follow up questions, they were asked what topics they had studied in this math class that they had used in another class.

Some of these students were able to come up with topics that they thought were useful, but some students were not. Useful topics included charting graphs, solving equations, trigonometry, finance, and probability. However, over half of the students (12) were either unable to think of any topics that would be useful, or thought that only the basic skills of addition, subtraction, multiplication, and division were useful.

Those students who believed that what they were studying did not apply to a world outside of school were also the ones (in general) who indicated their level of enjoyment to be 1 or 2 out of 5. Of the 12 students who could not think of anything other than basic skills that were useful, seven of them selected 1 or 2 out of 5 as their level of enjoyment of mathematics. One such student, during an interview, explained that she would probably enjoy mathematics if it were not for the algebra. When asked what she thought the purpose of high school mathematics was, she claimed to not know the purpose of topics other than finance (annuities, mortgages, and loans), and doing taxes. She claimed that knowing the formula for the circumference of a circle, and other such

formulas was handy, but other than basic skills, there was no purpose in the world outside of school for studying other topics in mathematics.

One other method of discerning whether students find the mathematics they study to be applicable, is to see if they recognize mathematics in other classes they take. Steen (2000) suggested that students do not recognize common mathematical tasks when they are in the context of other subjects such as chemistry or biology. Based on my research however, this does not seem to be the case with these M 33 students. Of the 21 participants, all but 4 were able to think of some mathematical skills that they used in other classes. Eight students claimed that they used algebra skills – mostly in Chemistry and Physics classes, and 6 students found that they only used basic skills. Interestingly, many students indicated that they used many of the mathematical skills they had developed in M 33 in Social Studies classes and in Career and Life Management class. These students claimed to have used their graphing skills when working with population pyramids, their probability skills, and their algebra skills. When this was reported to the teacher, she found this to be encouraging. Mrs. Stewart indicated that she tries to apply the mathematics in class to topics outside of the mathematics classroom, and had specifically targeted topics such as populations and world issues in Social Studies.

School mathematics and daily life

Generally speaking, these students believe that mathematics is important to society, but cannot see any direct importance of school mathematics to their own lives at this point. They see the use for school mathematics in other school subjects, but all of them found it very difficult to find a use, outside of school, for any of the topics that they

have studied in this course. This may be for a number of reasons, one being that the participants are not yet mature enough to know what they plan to do with their lives, and therefore cannot see where any of the topics they study in high school fit, or it could be because the topics do not relate to a world of everyday life outside of school, or perhaps their teachers have been unable to help them make these connections. Adults tell these teenagers over and over that education is so very important to their future, as it produces good, knowledgeable citizens, but it appears that the students cannot see many direct links to their future from their mathematics classes. As one student indicated on her questionnaire,

> *"They keep telling me how important math is later in life, but really I'm a labourer, so I just need to know how to figure out how [many] two-by-fours I need to build a 1650 square foot house."*

What the comments reveal

The comments and actions of these M33 students reveal their opinions and perceptions of the mathematics that they have studied in high school. Their comments can be summarized as follows.

- Math is not enjoyable. These students did not particularly enjoy mathematics class. Some did not enjoy mathematics class because they found it to be very difficult, while others found it to be repetitive and unchallenging. Many students commented on disliking mathematics class when there were lots of notes to take, or when they were to sit in their desk working on questions from the textbook.
- High school mathematics is of little use to the world outside of school. These

students found math class to be irrelevant to their daily lives, and to the lives that they hoped to enjoy in the future, except for the unit in which they studied financial mathematics (annuities, mortgages, and loans). Students indicated that they recognized mathematics in other high school classes, but found that they found it of little use outside of school.

For a comparison to these students' opinions and experiences, we can turn to those students enrolled in AM 30, and consider Mr. Rawding's students and their perspectives.

CHAPTER V: LISTENING TO APPLIED MATHEMATICS 30 STUDENTS

Now we turn to the case exploring students in AM 30. Recall that Alberta Learning intended this course to be one in which the approach to learning mathematics was mainly data driven, and relevance was a key issue. Students would only study mathematics in context, and would study algebra when it was needed, rather than for its own sake. The findings reported in this chapter are based on classroom observations, responses to the questionnaire, and interviews with selected students and with the teacher.

Mr. Rawdly and his Classroom

Mr. Rawdly has over twenty-five years of teaching experience, all with the school at which he currently teaches. He is a very highly respected educator in the jurisdiction, both by his teaching colleagues, and by the parents of the community, many of whom he has taught. He is also a very popular teacher among the students, as is demonstrated by the numerous times he has been asked by them to host the senior class banquet, and by the fact that mathematics students are always eager to get into his classes.

Mr. Rawdly's classroom is on the third floor of a relatively new school. It is a standard-sized classroom with a bank of windows overlooking tennis courts and a small green area between the school and a residential area of the city. The floors of the classroom are carpeted, and posters relating to mathematics and technology cover the walls. Mr. Rawdly's room is one of the few rooms in the school that still has a chalkboard (most others have white boards) and that is the way he prefers it. There is a teacher desk at the back of the room, along with tables covered with file boxes full of

materials, and a table at the front of the room, from which Mr. Rawdly teaches. Students' desks are arranged very neatly in rows of 6 students each.

Mr. Rawdly's teaching style is teacher-centred and relatively traditional. Most of the classes he teaches are centred around him demonstrating concepts on the board. Students participate by doing calculations with him and taking notes. Mr. Rawdly has a unique way of keeping his students on task by peppering his discussions with the names of the students in the class. For example, rather than saying "We need to find the square-root of this number", he will say "Geoff needs to find the square-root of this number, and Tracy tells him that it is x." when both Geoff and Tracy are members of the class. He also uses different accents and humorous ways in which to pronounce particular words (especially mathematics terminology) that the students find amusing.

Mr. Rawdly says he usually takes the majority of his class time to explore and explain concepts. Students usually get some time at the end of class to begin their homework, but they will, on the majority of evenings, have math homework. During the classes that I observed, one class was operated in a more contemporary manner, allowing students to explore concepts in groups and report back to the large group, and using group work to create a toy based on geometric shapes, but Mr. Rawdly admitted that this type of class was not common.

What the students said

Enjoyment and interest in mathematics

Again, participating students were asked to rate their level of enjoyment of mathematics on a scale of 1 to 5. A chart indicating the distribution of these ratings follows:

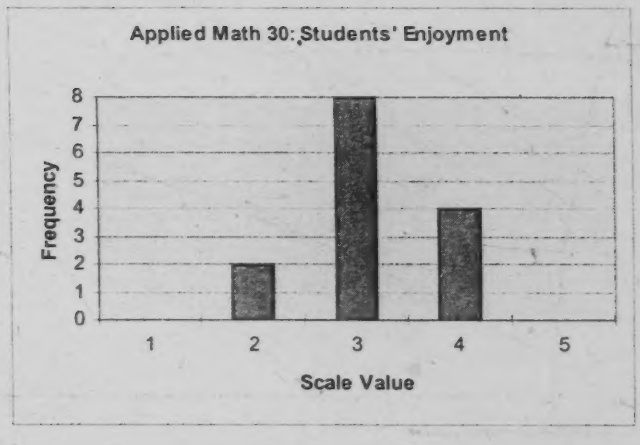


Figure 5: Applied Math 30 Students' Enjoyment Level

Of the 14 students in the AM 30 class who participated in the study, 29% rated their enjoyment of mathematics at 4 out of 5, and 57% rated their enjoyment level at 3 out of 5. Only 14% of these students indicated that they did not enjoy mathematics class. The mean rating from this class on this scale was 3.14 while the standard deviation was 0.66.

Those students who rated their enjoyment at a 4 out of 5, wrote comments that revealed how heavily their enjoyment of mathematics relied on their relationship with their mathematics teacher, or, like those students in M 33, on the difficulty level of the topic. One such student indicated:

> *I hate mathematics when it is trig or something new that I do not understand,*

While another student revealed how the subject got increasingly difficult for them:

> *I really liked math. I thought it was my best subject until high school when the difference between classes was too big of a difference.*

Some of the comments relating to their teachers indicated that the teacher was the only reason a student enjoyed mathematics. For example, one student wrote

> *I enjoy math this year more than any of my other years. I struggle in math and I don't feel like I need it, but Mr. R has a way of making you want to do good* [sic] *and listen. I guess it has something to do with the teacher you have.*

Other students indicated that their enjoyment of mathematics class depended on the teacher. One such student wrote,

> *It depends on the teacher. If the teacher doesn't try to have fun with it, neither will I. But if the teacher makes it interesting then I don't mind math.*

Other comments relating to teachers indicated that mathematics was enjoyable until the student experienced a particular teacher. For example, one student who rated his enjoyment of mathematics as a 4 out of 5 wrote,

> *With one of my teachers in grade 10, I felt that I wasn't really taught to the best level that I could have been. I retook the course.*

Another indicated that he enjoyed mathematics until they reached junior high, where he encountered a teacher who "*did not explain the concepts in math well enough*" which made it difficult for him to follow along with the material.

More students rated their enjoyment, or lack thereof mathematics based on their relationships with their teachers than they did on the difficulty level of the material. I found this interesting, and thought that it could be explained by a number of factors. First, the majority of students involved in the study truly enjoyed having Mr. Rawdly as a mathematics teacher. They found him to be dynamic and funny. These students enjoyed coming to his class because of these characteristics. It was evident to me that many of these students found it difficult to separate their enjoyment of Mr. Rawdly's teaching

style from their feelings about the subject he taught.

Many AM 30 students mentioned only disliking mathematics class when they did not enjoy the instructional style of a particular teacher they had experienced. I found these students' comments on their relationship with Mr. Rawdly, and their lack of comments on any unique relationship with any other teachers, whose teaching style they did enjoy, very interesting.

The two students who indicated that they did not enjoy mathematics related their dislike to math not being very social. One such student indicated that he only remembered this class, and that he found it to be boring, while the other student indicated that she used to like math, but now did not:

> *I used to like math when I was younger and just began learning it. We used to play games that related to math that were fun, and you learned lots at the same time.*
>
> *How hard do I work? How much ability do I have?*

Level of enjoyment of mathematics was compared with the student's perception of how hard they worked in math class, and their perceptions of their mathematical ability, and finally, to which of these two aspects they could attribute their success in mathematics. Students rated their work habits and their mathematical ability on a 5-point scale, with 1 representing a low level of work or mathematical ability, and 5 representing a very high level of work or mathematical ability. The mean score of how hard they worked on a scale from 1 to 5 was 3.54 (SD = 1.00).

Those students who were interviewed were asked to clarify what exactly they did in math class that showed that they worked hard, or what they would do differently if they did work hard. The students' indications of their ratings of work effort are shown in

figure 6 below.

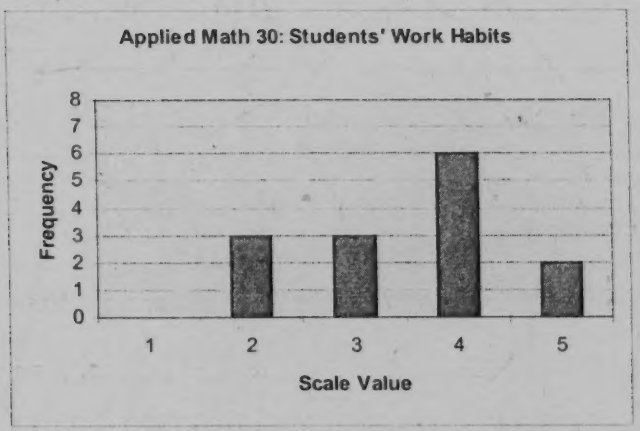


Figure 6: Applied Math 30 Students' Work Effort

When comparing students' level of enjoyment to how hard they worked, I was unable to find a correlation between these two values. There appears to be no relationship between students' enjoyment of mathematics and their perception of the difficulty of the course. This may be because of the large number of neutral responses to the question regarding enjoyment level of mathematics class.

Although some students had related their previous enjoyment or dislike of mathematics to the difficulty level of the material, it did not seem to play a part in the enjoyment of this particular math class. This could be because the students did not find this class particularly difficult, and therefore difficulty did not become part of the issue with this group. Half of those students who participated in the study earned an honours mark on the diploma examination, only two students earned a school-awarded mark below 50%, and one of those students scored well enough on the diploma examination to pass the course.

I then compared students' enjoyment of mathematics to how hard they perceived

that they worked. The mean indication of the students' mathematical ability on a scale from 1 to 5 was 3.25 (SD = 0.70). A graphical representation of the students' indications of their mathematical ability is shown in figure 7 below.

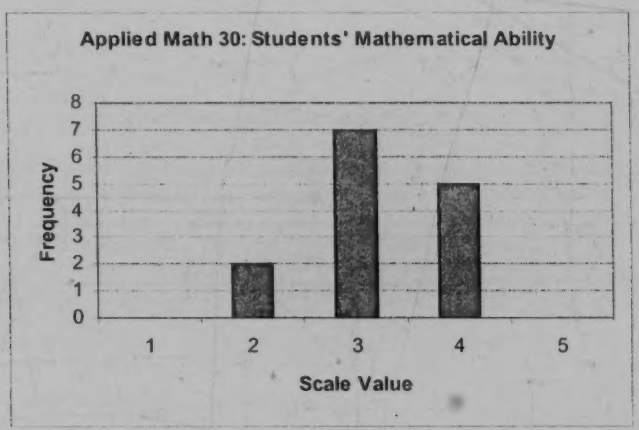


Figure 7: Applied Math 30 Students' Mathematical Ability

When comparing students' enjoyment level of mathematics to how much mathematical ability they thought they had, I found a Pearson product-moment correlation (r) of 0.66, a correlation that is acceptable at the 1% level of significance ($t = 3.07$, $N = 14$, $t_{12, 0.005} = 3.06$). This would indicate that students who rated their level of enjoyment higher also rated their mathematical ability higher. This is comparable to the SAIP (2003) findings, and to Ma and Kishor's (1997) study in which they found a weak yet positive correlation between enjoyment and ability.

These results indicate that the students, as a whole, feel that they enjoy mathematics, at least most of the time, that they feel they have fair mathematical ability, and that they work fairly hard at math class. This is consistent with their teacher's impression of this group as well. He felt that they were a hard-working group with ability suited to the applied mathematics course sequence. Whenever I was in the classroom, I

perceived the students to be willing to participate in math class and generally happy to be there.

During interviews, students who indicated that they thought they did work hard in class were asked what that meant to them in particular – what kinds of things they did that showed that they did work hard in class. All of the interviewed students gave similar answers – working hard meant that they studied, did all of their homework, asked questions in class, and reviewed (or at least attempted to review) on a regular basis.

The majority of students who participated associated enjoyment and interest in topics of mathematics to how applicable they were to their lives outside of school. As one student wrote,

> *I enjoy practical ones, such as finance. Little-used math isn't very interesting to me; I'd rather learn things I can expect to utilize.*

Another student wrote

> *I enjoy learning mathematics that will help me in later life – something that I can use everyday. It seems more worth my time and effort.*

The majority of students who indicated specific topics in AM 30 that they enjoyed the most, related their comments to the finance unit where they studied topics similar to the annuities, mortgages and loans unit in M 33. Other students indicated specific units or fields of study like spreadsheets and vectors. One student indicated that he found sinusoidal curves interesting because they related to sound waves. This particular student also wrote that he liked activities that related to his physics class.

Still other students did not mention any particular topic, but indicated that they enjoyed most activities in which many people were involved – activities like projects and other group work. For example, Lynn wrote that she liked it when everyone in the class

was involved in the actual lesson or teaching of the topic because it made her pay more attention and stay on her toes. Four students indicated that they liked lessons or activities of this type. Another student wrote that he enjoyed doing the projects because they made him research and do a lot of digging for information.

These students indicated that numerous activities they did in math class were interesting because of the teacher. As one student indicated, "*Having a teacher with a sense of humour helps. It makes the atmosphere less stressful.*" None of the students who participated mentioned any particular class or activity that they had done that they remembered or found interesting. Like the students in M 33 who wrote that they remembered a class period in which there was a coordinate plane drawn on the floor, I would have expected some of these students to remember a particular project or activity that they had worked on, but this was not the case.

There seemed to be no common topics or activities that the majority of students indicated were uninteresting or that they did not find enjoyable. Four students indicated that they did not enjoy working with the technology in math class – mainly the computer spreadsheets. These students stated that they found entering and manipulating formulas in spreadsheets time-consuming and confusing. One student however, wrote that she enjoyed knowing about mortgages and financing, but hated the fact that it was done on a computer. Four other students indicated that they did not find the probability unit interesting. Three of these students said that this was because they found it to be too difficult, and one said it was boring because it was too easy.

Rather than referring to any particular topic or field of study, some students indicated that they did not enjoy working on their own, from the textbook, or lessons in

which the teacher talked for the majority of the class. Similar to the M 33 students, these students wanted to be actively involved in the lesson. Many students gave general answers that indicated that they did not enjoy studying mathematics that could not be related to their worlds outside of school – math they considered to be overly theoretical. Kyle, for example, wrote, *"If I wanted to know why math is how it is, I would be in a pure level course."* Allison indicated that she could not understand what algebra had to do with things in life.

I was able to observe one class period in which students were working on a project in small groups. They were to design a kit within given parameters that included three pieces of basic plastic shapes for building three toys. Students were required to determine the amount of materials needed to make the pieces and the costs for manufacturing the pieces and the packaging. Students worked in groups of three people. Excerpts from my field notes follow:

> *Mr. Rawdly wanders from one group to the next asking questions of each – what kinds of designs can you make with the basic shapes you are using? How will you find out how much this will cost? Etc. The majority of students seem to be involved in the lesson.*
> *One student, Keenan, is not participating in the group discussion. Mr. Rawdly speaks to him quietly, and tells him to help out with the work. He does for about a minute, and then stops. Finally, Allison, one of his group members, scolds him for not helping out. He knows that if she goes to Mr. Rawdly and tells him that Keenan is not doing any of the work, Keenan will get a zero on the project, and the other group members will get the grades. Keenan begins to calculate the surface area and volume of some of the shapes they will need. This is the third class I have observed, and this shows the most amount of work Keenan has done in any of them. He tells me later, "I can't afford to get another zero." Interestingly enough however, is how Keenan responded to his questionnaire: he indicated that, on a scale from 1 to 5, the amount of work he did in math class was between 3 and 4.*

Of specific interest to me in this particular incident was how Mr. Rawdly's influence

seemed to be minimal, yet the influence of a peer seemed to carry significant weight.

I interviewed 6 students, and asked each of them which of two lessons they preferred, this one, or a lesson on vectors in which they were working with trajectories of planes. Five of those students indicated that they preferred the toy lesson. These students claimed that they enjoyed working with other group members to check to see if they are on the right track, that they could see the relevance of this project, and some admitted that it was kind of fun to build toys such as these. The one girl who said she preferred the vectors lesson claimed to have difficulty with spatial skills needed in the toy project. Because of this, she said she could relate more easily to the vectors lesson.

Why am I in this class?

Students were asked why they were registered in AM 30 rather than any other math courses offered at the high school like PM 30 or Mathematics 24. Half of those who participated indicated that they were in this course because they needed a grade 12 level math class and they felt that PM 30 would be too difficult. Five students indicated that they needed AM 30 to go on to further study. One student wrote, "*You can still get into college with Math 30 Applied and I thought it would be more practical for me to get an okay mark and relate it better to stuff I will use in life.*" Kyle indicated that AM 30 suited his interests better than did PM 30. Unlike the M 33 students, at least some of these students were making choices based on needs and interests.

Will I take another math course?

To further check students' interest in and enjoyment of mathematics, they were asked if, given the opportunity, they would ever take another mathematics class. Of the 14 participants, 5 (36%) indicated that they would take another mathematics class if they

had an opportunity to do so. All of the other students gave responses indicating that they would if they had to, but that they definitely would not prefer to take any more classes. For example, Tim indicated, *"I would rather not take any more math courses because I do not plan on doing anything that involves intense math."* and Maureen wrote, *"No, I probably wouldn't take further courses in math because I feel what I know now is enough to get me through life."* Although some students indicated that they would prefer not to take any more math courses because they struggled with the subject, none of them claimed that they would not like to because of an intense dislike of the subject.

A positive relationship with the teacher, and active participation in class

Relationship with a teacher

The majority of AM 30 students who participated in this study indicated that they enjoyed studying mathematics. For many of these students, their enjoyment level relied on the positive relationship they had with their mathematics teacher. This was not a theme that I expected to emerge from this study, but it became very clear to me through the students' responses to the questionnaires, the responses to the interview questions and their interactions with Mr. Rawdly during class, that most of the participants thoroughly appreciated and enjoyed Mr. Rawdly's teaching style and his personality.

Active participation

These students indicated that they enjoyed mathematics class more since they started taking applied mathematics courses because they were able to be more actively involved in each lesson. They enjoyed working in groups and discussing the work with their peers. As one student said during an interview, *"When we do projects, we can*

research and we can work with other people – I find that easier."

When asked to recall a time that they did not enjoy mathematics, most related memories of mathematics classes in which they found the teaching practices to be uninteresting, while some related times where the subject matter became too difficult for them. Three students indicated that they moved from the pure stream into the applied stream because they found pure mathematics to be too difficult for them, and that they liked applied better because the teacher did not seem so rushed to complete the material.

Importance and Relevance of Mathematics

All of the participants in the study were asked about the importance of mathematics, both to them, and to society in general. As a follow-up, they were also asked which topics they had studied in this mathematics class they used in other classes.

All but one of the participants indicated that they thought mathematics was important to society. That one participant thought that math is not used everyday, other than for buying cars and houses, and determining net pay. Surprisingly, however, she rated her enjoyment of mathematics as 4 out of 5. All other participants believed that mathematics was very important to society. Many of these students related it only to financial matters, but others were able to express an understanding of the mathematical basis for architecture, economics and electronics. One student indicated that mathematics was important to society because it added to everyone's basic knowledge.

I also inquired as to how important mathematics was to each individual student. Four of the participants believed that mathematics was not important to them. These students predicted that they would not require any of the mathematical skills they had

acquired in high school in later life. One student indicated that she did not want to have anything to do with mathematics or science because those two subjects did not interest her. Another student indicated that she did not think that mathematics, past the basic skills, was important.

The remainder of the students believed that mathematics was important to them. However, many of these students claimed that mathematics was important because they needed it to get into a post-secondary institution, not that they would actually require the mathematical skills that they had learned. One student indicated that mathematics was important because it is important to use one's left-brain. Most students however, felt that mathematics was important because they would need it in their financial dealings. Those students who were interviewed however, were prodded further about the importance of mathematics. At this point, they were able to discuss the necessity of mathematics to such things as architecture and electronics, but claimed that since these topics were of no interest to them, they were not important.

To further explore the relevance of mathematics to their daily lives, students were asked if they had ever used any of the mathematics that they had studied in this class in any other class. Most of these student indicated that they had used the graphing and algebra skills that they had developed in science classes. One student mentioned that they had used their skills in Career and Life Management class. Three students claimed never to have used any of their mathematical skills in other classes. Two of these students also did not believe that mathematics was important to their personal lives outside of school. The third student indicated that mathematics was important to him, but only because he needed it to get into college. It was not immediately evident whether the students were

unable to recognize the mathematics they studied in AM 30 as it appeared in other classes, or whether there were no such applications.

The World Outside of School

The applied mathematics stream was designed to allow students to see the importance of mathematics in their lives and in their world around them. It seems, at least with this group, that students are able to see the relevance of mathematics to courses other than mathematics in school, and outside of school in such areas as financial applications, while some students were aware of architectural and electronic applications as well. However, they seem unable to see themselves using mathematics (other than basic arithmetic) on a regular basis. They are aware of how important mathematics is to society, but seem to believe that advanced mathematics relates mainly to other people in society, not to themselves.

What the Comments Reveal

The comments and actions of these AM 30 students reveal their opinions and perceptions of the mathematics that they have studied in high school. Their comments can be summarized as follows.

- For the most part, mathematics class is enjoyable. These students claimed to find mathematics class to be rather pleasant either because their teacher made it so, or because the material they studied was manageable. Activities that were most enjoyable were ones in which they could work with other students and research topics of interest, and how mathematics was related to them.
- Some of these students made the choice to take AM 30 because they perceived it

would be useful to them, rather than because they thought it was easier than PM 30, or because of restrictions placed on them by post-secondary institutions. However, although most of the students believed that mathematics was important to society, many were unable to see how it was important to themselves.

For a more detailed comparison of the comments from the M 33 students and the AM 30 students, we turn to the next chapter.

CHAPTER VI: PUTTING IT INTO PERSPECTIVE

What the students said

Enjoyment of mathematics class

The research literature suggests that the affective domain must seriously be considered when deciding on curriculum and on topics to study (Maher, 1982; Neale, 1969; Ma and Kishor, 1997). Given the responses of the students involved, this study supports this claim. For both classes, AM 30 and M 33 students were asked to indicate their level of enjoyment of mathematics class on a Likert scale of 1 to 5 (1 being not enjoyable, and 5 being very enjoyable). No student indicated their enjoyment level as a 5, and students in M 33 indicated that they enjoyed mathematics less than those students in AM 30. There were no differences in ratings between males and females in AM 30, and only a slight difference in M 33, with three females (3/7) indicating a 1 as their level of enjoyment, while all males (7/7)indicated values between 2 and 4. In M 33, the number of students who indicated that they did not enjoy mathematics was slightly less than half of all of those who agreed to participate, while 19% indicated that they did enjoy mathematics class. In AM 30, 14% of those who chose to participate indicated that they did not enjoy mathematics class while 29% indicated that they did enjoy mathematics. In both cases, the remainder of students gave neutral responses.

Other literature suggests that many students disengage from school mathematics because they find the subject as taught to be uninteresting and irrelevant or not of much use to them (NCTM, 2000), or they are confused about the complexity and precision of the subject (Turner et. al, 1998). The reasons that participants in M 33 gave for not

enjoying mathematics class seemed to parallel many of these reasons for disengaging in mathematics. In contrast, because many of the AM 30 students were more interested in math class, or found the course manageable, they subsequently did not seem to become as disconnected from the concepts that they study.

Both M 33 and AM 30 students claimed they liked mathematics when it made sense to them, and disliked math when it became laborious, abstract, and symbolic. One student in M 33 wrote that he enjoyed math before he progressed to grade 7 because that was when they started working with algebra, and he *"hated"* algebra. Both groups of students enjoyed playing games in class, having competitions, and doing investigations and projects with their peers.

Given what we know about human learning, these comments are significant. "[L]earning is …understood as participation in the world" (Davis, Sumara, Luce-Kapler, 2000, p. 64) and these students had a need to participate in the world of mathematics. They needed to become part of it rather than feeling as if they were observers who were expected to memorize irrelevant material. They disliked being lectured to, and found it to be of little use in their construction of mathematical understanding. They required social interactions, both between themselves and their peers, and between themselves and their teacher to develop a fit between their subjective knowledge of mathematics, and the socially accepted, objective mathematics (Ernest, 1991). Through these social interactions, the dynamic form of learning is created. The mathematical knowledge that each of these students becomes part of through mathematical activity enacts a change in the mathematics that they know, and that each other person involved, including the teacher, knows (Kieren, 1998; Davis, 1995).

The world outside of school

Mathematically literate citizens are imperative to society. There are multiple contexts in adults' lives that require numbers, quantities, mathematical concepts, and algorithmic processes. These contexts include personal finance, informed citizenship, social action, the work place, further education, and active parenting. To create such citizens students learning, and at least some of the mathematics that we teach must be within contexts related to the world outside of school.

Although "pure" mathematics teachers often rely on applied contexts as both a means to motivate students and as a source of problems, ultimately, the focus of mathematics is on abstract patterns. The context becomes part of the "irrelevant detail that must be boiled away over the flame of abstraction in order to reveal the previously hidden crystal of pure structure" (G. Cobb, 1997, p. 76). Mathematical literacy however, depends on context. Whether the mathematical patterns and relationships have meaning depends on the situation in which they are found.

The question of context was the most notable difference between the two groups. Most of the AM 30 students believed mathematics is important both to themselves and to society as a whole. These students could see the usefulness and need for the mathematics that they learned in school in financial situations, and in interpreting data, but many of them could also understand the basic mathematics underlying principles of electronics and architecture. The M 33 students however, believed school mathematics is important to society, but not necessarily to themselves as individuals. They knew that it is important for some people to know mathematics but most of them could see no place in which they would use the mathematics that they had studied in high school. Over half of these

students indicated that they found no topic studied in math class, outside of the basic skills of addition, subtraction, multiplication, and division to be of any use outside of school. In so much that the students could not see much use for these courses outside of school, they could however, see many references to the topics they studied in math class in other classes.

Based on the findings of this study, it appears to me that those students enrolled in AM 30 were much more engaged in the study of mathematics than the students in M 33. For the most part, AM 30 students appeared to be interested in knowing where the mathematics they learned could be related to their worlds and to the lives of other members of society. In contrast, the M 33 students appeared to believe that mathematics was uninteresting, and could see no purpose for it beyond the walls of their high school. If we consider this difference in terms of a social constructivist framework, it would appear that the social interaction through projects, group-work and investigations that seems to be much more apparent in the AM 30 class may have resulted in a greater engagement of the learners with the mathematics.

Success in mathematics

Individuals who attribute successful academic performance to effort rather than ability or who have positive self-perceptions of performance generally try harder and persist longer at difficult or challenging tasks (Dweck, 1986; Lich & Dweck, 1984). These students, according to Sternberg (1985), are more likely to succeed than those with negative perceptions.

All of the students who participated in this study have experienced success in mathematics by reaching a grade 12 mathematics class. Of the 14 AM 30 students who

participated in the study, 7 indicated that they attributed their success in mathematics to hard work rather than to their ability. All of these students claimed that mathematics was not a subject area in which they felt more confident than other subjects, but that they had succeeded in getting to AM 30 by doing homework and preparing for exams. On average, students who believed they owed their success in mathematics to hard work rated their enjoyment in mathematics only slightly lower than those students who indicated that they attributed their success in mathematics to their ability rather than any extra effort they had put in rated their enjoyment level.

Many of the students who indicated that they had experienced success in their mathematics course because of their abilities, indicated that they believed they had done so because they felt they had covered much of the material in other classes. Many of these students indicated that they had taken PM 10, PM 20, and then switched into AM 30. Because the rigor between these course sequences is significantly different, these students may have discovered that they no longer had to work as hard as they had in the past in order to succeed. Generally speaking, all of these students enjoyed mathematics class, some because they found it to be significantly less demanding than previous courses, some because they enjoyed the teacher, and some because they enjoyed the explorative, investigative nature of the course. These were all motivation factors in their success in mathematics.

In contrast, of the 21 M 33 students who participated, 13 indicated that they attributed their success in math class to hard work, while 6 indicated that they attributed their success to ability. Again, there was a minimal discrepancy between the two groups as it relates to their enjoyment ratings of mathematics class, with the students who

attributed their success to hard work rating their enjoyment level only slightly higher.

The purpose of asking students to what they attributed their success was to test the hypothesis that students who indicated that they worked hard in math class would experience more success, and thus, enjoy mathematics class more than those whose mathematical ability was the reason behind their success. The flaw in the question as it turns out was that all of these students have experienced success in mathematics by reaching a grade 12 mathematics course. To what they attribute their success can not be so easily determined by answering one question, because of the complexity of the concept of success. Many of these students in fact believe that they have not been successful in mathematics class, simply because they are enrolled in AM 30 or M 33, rather than PM 30.

Relationship with the teacher

The students' relationship with their teacher was not an area I originally intended to study. However, it became an issue after reviewing the comments on the questionnaires from those students enrolled in AM 30. These students indicated a strong, positive relationship with their teacher, while students in M 33 made no mention of the importance of a positive relationship with the teacher. I became curious as to how this could relate to the student enjoyment of mathematics and mathematics class.

AM 30 students seemed to rely heavily on their relationship with their teacher when deciding on their feelings about mathematics class. As one AM 30 student wrote: *"It depends on the teacher.... If the teacher doesn't try to have fun with it neither will I; but if the teacher makes it interesting then I don't mind math."*

Although the enjoyment level of mathematics by those students enrolled in AM

30 may have been overly influenced by their relationship with their teacher, the findings are still important. My conjecture is that these students are able to enjoy the relationship they have with their teacher because they find the mathematics they are learning manageable and relatively inviting. If the subject matter they study is not so difficult as to cause them to develop a dislike for it, then they maintain the enjoyment of mathematics that many claimed to have had when they were in elementary school. At worst, these students may simply maintain indifference toward the subject. As long as the material remains manageable, students' feelings toward mathematics will not change unless another factor is introduced – that of their relationship with the teacher. The possibility of the teacher having a greater influence on students' enjoyment of the class when the difficulty level of the course seemed more in line with the students' ability would explain why there was a higher percentage of AM 30 students who based their enjoyment of mathematics on their relationship with the teacher than there were M 33 students who indicated so. Students in M 33 generally found mathematics to be very difficult and therefore they did not enjoy it. Even a very strong relationship with a teacher whose style they thoroughly enjoyed could not erase their feelings about the subject matter.

Students, it appears, feel more relaxed and able to develop a positive relationship with their teachers if the topics of study do not provide situations in which the students experience a great deal of anxiety. I wonder if students and teachers are unable to develop a relationship if there is no time to discuss issues or to explore topics.

On more than one occasion, both of the teachers involved in this study made comments that related content and rigor to comfort level of their students. In a follow-up discussion, Mr. Rawdly indicated that he found that all of his AM 30 students seemed to

enjoy mathematics class more than the M 33 students that he had taught in the past. He thought that it had to do with the conversational tone of the course. He indicated that he felt students were more comfortable with the introduction of topics in AM 30 because they all had some previous experience with aspects of them. At the introduction of each new topic, students could participate in a discussion of how they perceived the topic related to society in general, or their life experience specifically.

Mrs. Stewart made similar comments, stating on one occasion that, from her M 33 students' perspectives, mathematics did not seem to be anything more than a course that was required in order to leave high school. As was evident from the lessons that I observed, Mrs. Stewart understood very well the need to relate topics to students lives outside of school, but due to the required content, many of her attempts became short-lived or contrived. The topics, in their current level of rigor, were too superficial to be applicable, and so to include them, the context had to become simplified to the point of being contrived. Mrs. Stewart was unable to make comparative statements about students in M 33 and those in AM 30 because she had not ever taught AM 30.

The themes that emerged from the comments of these participants generally related to four areas: enjoyment of math class, the relevance of mathematics, success in mathematics, and the relationship with the teacher. For an indication how these themes relate to the larger picture of the purpose of high school mathematics, and the roles of the school jurisdiction, the educator, and policy makers, we turn to the final chapter.

CHAPTER VII: CONCLUSIONS

The purpose for conducting this research was to listen to the voices of the high school mathematics students with respect to their beliefs about, attitudes toward, and experiences with mathematics and learning mathematics. Reform in mathematics education in Alberta has been subjected to the voices of public officials, teachers, and parents, but has rarely, if ever, been influenced by the voices of students. By listening more carefully to these students, I hoped to better understand their experiences in AM30 and M33, given the differences in curriculum and approaches to learning that these two programs support.

The applied mathematics program had a rather unfortunate introduction into Alberta high schools, during which time public officials, teachers, and parents all expressed opinions and concerns about mathematics education. However, some teachers claimed to observe a new outlook from their students, with regard to the usefulness of mathematics, and their beliefs about learning mathematics, compared to the students these teachers had taught previously. My research then, centers on a question of whether or not, as suggested by these teachers, research can demonstrate a link between students' attitudes, beliefs, and perceived usefulness of mathematics. My research question, "In what ways do students' attitudes and experiences compare, given the contexts of AM 30 and M 33?", is approached by attempting to answer the sub-questions

- What importance and relevance do students place on mathematics?
- What aspects of math class do students enjoy or not enjoy?
- To what do students attribute their success in mathematics?

- Do students consider mathematics important to society or to their lives?
- Do students recognize where they use, or could use the mathematics that they have studied?

To inform these questions, I decided that the most efficient method would be through collecting both quantitative and qualitative data. I selected a M 33 class in one school, and an AM 30 class in a neighbouring school, and I collected data through questionnaires, interviews and observations that helped to describe students' attitudes toward mathematics; some of their experiences that may have lead to those attitudes; and the aspects of mathematics class that students said they enjoyed or did not enjoy. This was done in an attempt to not only report the experiences of students in these two programs, but also to interpret and explain, as best as I could, the significance of some of these experiences.

The questionnaire contained items that asked students to indicate their views and preferences using a Likert scale, and short-answer questions that asked students to describe some of their experiences in greater detail. Classroom observations were conducted in an attempt to identify students whose actions did not appear to match their views about mathematics and mathematics class as expressed on the questionnaire, and also those whose actions did appear to match their opinions. A selection of students whose opinions did seem to match their actions, and those whose opinions did not seem to match their actions were selected for informal interviews. These methods of data collection lead to some of the data being quantitative, while most remained qualitative in nature.

Important, Relevant, and Significant Findings

Students in AM30 indicated that they enjoyed mathematics more than did those students in M33. There were no differences in enjoyment level of mathematics class between males and females in AM30, and only a slight difference in M33. In M33, the number of students who indicated that they did not enjoy mathematics class was slightly more than half of all those who participated (11 out of 21), while 20% of AM30 students indicated that they did not enjoy math class. Conversely 19% of M33 students indicated that they did enjoy mathematics while 27% of the AM30 students indicated that they enjoyed mathematics. The remainder gave a neutral response.

The reasons that participants in this study gave for no longer enjoying mathematics class, and disengaging from it seem to parallel many of the findings of the research literature. They claimed, like NCTM's (2000) findings, that the subject was uninteresting and irrelevant, or, as Turner et. al (1998), they claimed to find the course too difficult. It appeared however, that because the AM30 students were more interested in the topics they studied, they remained more engaged in the subject.

Many M33 students indicated that they enjoyed learning mathematics when it was easier to understand, and for most of them, that was before the introduction of algebra as a topic to be studied. In general however, AM30 students did not claim to find the course they were taking to be too difficult. Only 33% of the students made comments that math had become too difficult for them to understand, and of those, many of their comments related to their attempts to complete courses in the pure stream before switching to the applied stream.

The question of context as it relates to the topics of study was the question in

which the responses between M 33 and AM 30 students were the most different. Most AM 30 students could see the usefulness and need for mathematics that they learned in school (especially as it related to financial situations, and in interpretation of data). M 33 students however, believed that school mathematics was important to society, but not necessarily to themselves as individuals. These students indicated that they understand how important it is for some people to understand mathematics, but most of them could see no place in the world outside of school in which they would use the mathematics that they had studied.

One of the most prevalent themes that emerged from the students' comments was a need to be actively involved in the lesson. Students who commented on classes that they strongly disliked described situations in which they felt they were forced to sit and listen to lectures. When asked to recall specific math classes or topics that they enjoyed, they commented on classes in which they could work in groups, or explore topics through projects, or other extended tasks. Any kind of novel approach to teaching mathematics seemed to strike a positive note with these students.

The Purpose of High School Mathematics

Those selected students from each class who were interviewed were asked what they thought the purpose was, for teaching a course such as the one in which they were enrolled. The M 33 students who responded gave responses that very closely paralleled the following

- The financial applications were to prepare them for work and life
- The course was to give those students in grade 12 who did not like math, a grade

12 mathematics course

- There is no recognizable purpose for this course

I believe that these students had not seriously considered this question in the past. My conjecture is that they do not really know why the content of this course is as it is, other than because a person or persons in a position of authority believe the topics to be of importance.

Those students enrolled in AM 30 gave comments that indicated that they understood that the purpose of the course they were taking was to show where the mathematics they knew could be seen in the world outside of high school. These students gave answers similar to the following:

- To deal with financial situations;
- To show different ways of solving equations and graphing, and working with numbers;
- To learn mathematics that you will need for careers like engineering and others related to computer technology.

If we refer to the purpose of the two mathematics courses, as they are stated in the respective program of studies, it becomes clear that the intents of the programs are different. So, we must consider: Is the purpose of high school mathematics in Alberta to teach abstract, formal thinking in a logical progressive manner, or is it to teach mathematical understandings of situations that arise in every day living? Alternatively, is the purpose two-fold?

The purpose of the second-stream mathematics courses at the high school level is one that has been grappled with by provincial policy-makers. It is obvious that people

who worked closely with the M 33 program believed that the purpose was two-fold. It was important for students to learn algebraic and graphing techniques, and hence topics such as quadratic functions, and solving equations were mandated. However, topics such as financial applications and trigonometry were also included.

Those involved with the AM 30 program (some of whom were also involved with M 33) appeared to have had a slightly different opinion of the purpose. This assertion is based on the fact that there is virtually no algebraic component to AM 30 outside of linear algebra, and most topics that are mandated are directly related to a world of citizenship and employment. Topics of study include statistics and probability, financial applications, trigonometry, and design. It seems that these curriculum designers would be in full agreement with Noddings (1994) who claims that "one should not need a statistical study to be convinced that most people use virtually no algebra or geometry in their personal or work lives" (p. 90).

It seems a relatively simple decision that the second-stream mathematics courses in high school should concentrate on teaching students how and where they can apply the mathematics they know in a world in which they can become responsible, productive citizens. If these students are not intending to continue study in a science or business-related field, they will have little or no use for the formal mathematics that they may study. The problem with this decision is that it has virtually closed the door for many of these students to any kind of post-secondary education.

The role of the universities, colleges, and technical schools

Many post-secondary institutions in Alberta do not accept AM 30 or M 33 as an acceptable course for admission into most faculties or departments. Generally speaking,

the mathematics departments at these institutions were concerned with the lack of rigor of mathematics in M 33 and were very concerned with the lack of algebra in AM 30. So strong were their concerns, that University of Alberta accepts Drama 30, or a second language at a 30 level in places where neither AM 30 nor M 33 are accepted, regardless of the fact that neither of those subjects has a diploma examination, and both AM 30 and M 33 do (University of Alberta, 2003).

The question of whether or not we should be teaching courses that are not accepted for entrance into post-secondary institutions to students who may or may not have finalized their future plans is one that should be considered. However, it is my opinion that as long as there are two mathematics courses offered at the grade 12 level, where one is considered to be more rigorous than the other, post secondary institutions will most likely never accept both. Many people involved in high school education consider the admission departments at these institutions to use PM 30 as a screen for admission. Institutions do not have to create entrance examinations if their admission requirements are such that less than half of Alberta students are even eligible. This alleviates an issue for post-secondary institutions and instead puts it in the laps of Alberta high schools.

Until post-secondary institutions recognize the virtue of mathematical thinking that has not necessarily been formalized into algebraic symbols, the future of initiatives such as M 33 and AM 30 will forever be doomed to a second level of academics. Students who become more and more convinced of the need for a post-secondary education will recognize that these courses will not get them there, and will end up back in the pure mathematics stream.

The irony in this scenario is that it has been my experience with high school mathematics students, that many of those who do not study the formal algebra as seen in the pure sequence develop strong analytical skills, and become much better problem-solvers than many who study formal algebra, and even calculus. The students I have had experience with who study formal algebra, are convinced that algebra is the only way in which to approach a problem, and are unable to reason their way through many mathematical situations.

Rarely are citizens faced with problems that are presented in the language of algebra. Most often we need to think mathematically about situations laden with incomplete data, ambiguous graphs, uncertain influences, and hasty generalizations. A formal study of algebra is of little use in such situations. Perhaps it is of importance then, to consider that many students at post-secondary institutions do not necessarily require the formal algebra skills to have success, but rather require the analytical, problem-solving skills that those who study applied mathematics seem to develop.

Implications for Schools and School Jurisdictions

The role of the jurisdiction

The goal in a constructivist classroom is for students to make conjectures and to formulate questions. This takes time and instructional coaching to develop. In my experience, without appropriate instruction and models students tend to note only surface features, and either ask superficial questions or no questions at all. If only the surface of a topic has been explored, students are never able to internalize their learning. This causes a lack of retention of the mathematical knowledge. Immediately, the entire basis for

constructing knowledge is destroyed. Without an adequate base, there is no foundation on which to build new knowledge.

The students in this study wanted to be active participants in their learning. They did not enjoy lessons in which the teacher lectured for the majority of the time, and most of them who had experience with projects and group-work found that to be more interesting and stimulating than the more traditional lessons. In order to enable students to be active learners, teachers must design lessons and problems around issues that stimulate questioning and inquiry. According to Carole Greenes, these problems should be relatively novel, have some connection to students' past experiences, have multiple solution paths, and should be in contexts that are interesting to students (Greenes, 1995). Obviously, these problems require a significant amount of time for teachers to prepare. Given a curriculum, in which many teachers believe a new topic must be "covered" every day, this may be impossible.

For this reason, the actual teaching load per term must be reconsidered. It is interesting to me that the applied mathematics course (one which requires a significant change in teaching style) was more readily adopted, even during the optional implementation phase, by school divisions in which teachers have less of their work day dedicated to actual teaching, and therefore more time for lesson preparation. All of Calgary Separate high schools, and most of Calgary Public high schools implemented AM 30 during the optional implementation phase, whereas only one school in each of Edmonton Public and Edmonton Separate adopted the program. Calgary teachers have less of their day dedicated to instructional time than those teachers who work for either of the Edmonton boards.

The role of the educator

The students involved in this study clearly indicated their preference for mathematics lessons and topics that were related to their lives outside of school, that they believed to be useful for their future, and in which they could be actively involved – either by working with other students or investigating problems.

Mathematics becomes useful to a student only when it has been developed through a personal intellectual engagement that creates new understanding. Much of the failure in school mathematics is due to instruction that is inappropriate to the way most students learn (National Research Council, 1989). Students simply do not retain what they learn from lectures, worksheets, or routine homework for any significant time.

Regardless of the content prescribed in the curriculum, teachers have an impact over the manner in which mathematical concepts are taught. Knowing where mathematics relates to lives beyond the classroom, and planning lessons that have students' actively involved are the teachers' responsibility. Since the participants in this study have clearly indicated how these types of activities are more enjoyable for them, and thus improve their attitudes towards mathematics, we as teachers must admit that we can no longer pretend that we can teach mathematics in a vacuum – beyond the influence of whether or not students find the material relevant or enjoyable.

The intent of the revised high school mathematics curriculum was to decrease the amount of lecturing in mathematics classrooms and to increase the amount of interactive, constructive learning. Research has shown that the teacher is the main influence on how a curriculum is delivered. "No other intervention can make the difference that a knowledgeable, skilful teacher can make in the learning process" (NCTM, 1998, p. 3).

Teachers need to focus on creating environments in which questions are encouraged – where both the teacher and the student are involved in one another's thinking and function as a mathematical community (NCTM, 1991).

The teacher is the one who must deliver the curriculum in the manner in which it was intended, and must encourage discourse between students, and between themselves and students in the classroom. The teacher must build and maintain an environment in which children can learn mathematics by relying on their lived experience and building upon it. Teachers need to model a mathematical community for their students. Many students associate mathematics classes with sitting and copying notes the teacher writes on the board. This is exactly the opposite of the view of mathematics classrooms in which students are involved in meaningful mathematical discourse.

So, what does it take to build such an environment? First the role of the teacher is changing. No longer is s(he) expected to be an expert, but rather a facilitator of learning. Second, teachers must possess the confidence to act in this role. Creating an environment where questioning is the norm requires a teacher to be very confident and comfortable in his or her subject area. This teacher must also be comfortable with allowing students to learn in different situations and in a different manner than other members of the class. Finally, teachers must have the time to build and maintain such an environment. Teachers require time to plan lessons and adequate class time for students to investigate the material.

Many of these criteria are the very ones that Alberta's mathematics teachers are currently lacking. Teachers will only change the way that they teach if they believe that their current methods are not working and if a new method brings immediate, positive

results. Many teachers in Alberta feel that their current methods are working, and working well. Alberta is nationally (and internationally) renowned for its successful mathematics students. We traditionally do well on international studies – recently scoring in the top third of the world on the Third International Mathematics and Science Study, (Unland, 2000) and our best students on the whole, go on to promising futures. The question arises then, why mess with a good thing? The answer is simply because this approach has worked well for a number of students, but it is certainly not the best approach for the majority of mathematics students.

Teachers must learn to develop rich problems – problems that engage students to see how mathematics arises naturally in many contexts. These problems do not necessarily have to be directly applicable to the world outside of school, but must encourage students to consider, and reconsider their thinking about the problem. One topic of study that was common to both the AM 30 and the M 33 courses was financial mathematics. I believe this topic, taught in the manner in which both Mr. Rawdly and Mrs. Stewart taught it, is an example of a rich problem, and that is why this particular topic came up over and over again when students were asked which topics they enjoyed the most, and which they thought would be the most useful to them in their future. Both of these teachers approach this unit using consumer problems – purchasing a vehicle, saving for college or university, buying a house, paying taxes, or investing money. These situations arise outside of the classroom as well, create opportunities for dialogue, and help students to become informed consumers. Both teachers also created scenarios, into which students were put, and used these scenarios to assess the students' learning.

Financial mathematics is a rich context because the topic itself, initially does not

appear to be strictly mathematical in nature (no angles, or algebra), because it has direct application to students' lives both now, and in the future, and because it is a topic that can be discussed outside of the classroom (with parents and peers) with a general degree of ease. These aspects, when considered together, create a meaningful, useful experience for students.

Many mathematics students may lose interest in mathematics and mathematical inquiry because there are few meaningful experiences on which they can build. Our classrooms need to reflect the belief that knowledge is not attained in isolation, but rather, knowledge is negotiated, and agreed upon by participants in the learning process. "Learning occurs when children's conceptions are challenged by more complex situations, different contexts, or by conflicting data or information" (Greenes, 1995, p. 86). Teachers must learn to use more techniques that build on this premise – working in groups to encourage social exchange and to develop mutually agreed-upon understandings of the mathematics, building mind maps or concept maps to organize students' thinking about concepts, or using data sets for concept-attainment or concept formation (Bennett & Rolheiser, 2001).

Changing teachers' beliefs and practices does not occur merely by placing new programs at their disposal, but also by initiating and guiding the development of their pedagogical understanding (Manouchehri & Goodman, 1998, p. 40). For all teachers, the presence or absence of progressive leadership is instrumental in their persistence to using new mathematics programs. Significant money and resources from Alberta Learning and school jurisdictions must be put into place in order to gain teachers' understanding and beliefs in this program. This change in philosophy will not come quickly, or without a

price.

Mathematical confidence is of great concern to teachers, parents and administrators. Many teachers are not prepared to answer students' questions with a response such as "I don't know, perhaps we need to investigate that question." In order to minimize the chances of this situation occurring, an insecure teacher may not develop an environment of inquiry.

To teach in a constructivist manner, teachers need to possess mathematical knowledge that is both broad and contextual. Although a lack of mathematical knowledge and understanding were certainly not issues that the two teachers involved in this study had to deal with, many teachers in Alberta have not the experience nor the expertise gained at university to be comfortable with the material that they must teach.

Mathematical knowledge is not the only area in which many teachers are lacking. There is a significant amount of pedagogical knowledge that is not available to many teachers. In a study conducted by Manouchehri and Goodman (1998) found that many teachers had a "lack of familiarity with appropriate questioning techniques and knowledge of multiple approaches to problems and investigations" (p. 38). Many teachers believe that constructivist teaching involves solely group work. However, this is "no guarantee that [students] will be learning mathematics or developing the cognitive processes that facilitate mathematical explorations, investigations, and learning" (Greenes, 1995, p. 91).

Recommendations

After listening to the voices of the M 33 and AM 30 students who participated in this study, a number of recommendations appear for me.

1. Teachers must receive adequate pre-service and professional development in areas of educational pedagogy. Only then, can they come to understand why many of their methods of instruction may not be working well. These teachers can learn about the constructivist framework, and, with the help of many of their colleagues, develop lesson plans and projects that support a constructivist learning environment.
2. Teachers, parents, and policy makers must consider the attitudes and beliefs of students in mathematics classes. Students do have a right to question why they are doing what they are doing, and it is our responsibility as professionals to explore the mathematics in context with them. Contextual mathematics must be brought into the mathematics classroom. Students find it difficult to relate the mathematics for which they cannot see a connection to the world outside of school. When these students fail to see this connection, they disengage from mathematics, and lose interest.

Implications for Further Study

Regardless of the mathematics we teach or to whom we teach it, we must be concerned with the students' perspective on it, and for what they believe it can be used. Students such as those in AM 30 and M 33 are not necessarily motivated by grades (actually many of them are simply pleased as long as their marks are above a passing grade), nor are they necessarily motivated by acceptance into a post-secondary institution. For this reason, we must develop other ways to motivate these students. My research suggests that one such way is to make the topics of study relevant to both the students' current lives and to the lives they hope to lead in the future. Mathematics class

should be a place where students have the opportunity to not only learn about mathematics, but how mathematics relates to the world around them, including art, music, finances, and patterns. Relative to this belief then, further study could be concerned with exclusively relating the mathematics that students study in school to the world outside of school. Teaching the same program of studies to comparable classes, but using two different approaches, and then considering their attitudes and perceptions of the usefulness of mathematics could yield interesting findings.

The students enrolled in AM 30 could recognize more mathematics that they had studied in school when they encountered it in the world outside of school than their counterparts in M 33. However, the students enrolled in M 33 could recognize more mathematics in the other classes they were taking than could the AM 30 students. This is an aspect of the study that I found particularly interesting. Further study could be done to explore this more thoroughly. Unfortunately however, M 33 is no longer offered in Alberta high schools, but I believe that this study could be considered by comparing the views of students in PM 30 and those in AM 30.

Final Thoughts

We as mathematics teachers can no longer continue to believe that mathematics learning is independent of who is learning and the environment in which it is taught. In fact, what we teach, and how we teach it are greatly affected by students' attitudes toward the subject and the topics within it, and their beliefs about the usefulness and importance of mathematics. We as teachers, parents, and policy makers can no longer choose to ignore the voices of the students our decisions directly affect. Through the questionnaire used in this study, I have listened to these students, and they have told me three main

things about high school mathematics education in Alberta.

1. Mathematics learning must be relevant to the lives that these students currently lead, and to the lives that they may lead in the future. Teachers and policy makers must make attempts to connect the mathematics at hand with the world outside of school.
2. Students must be actively engaged in the mathematics that they are learning – they do not learn well by listening to lectures and taking notes. They want to work on authentic tasks, and be able to work together with their peers on these tasks.
3. The prescribed content of the mathematics courses must be within the capabilities of the student. Many of the students in this study claimed that they found mathematics to be very difficult. However, this may be because they found it irrelevant, and thus difficult to remember. If the content continues to be relevant to students, then the concepts become easier to remember, resulting in higher retention.

In the first chapter of this thesis, I discussed the question, "When will I need this?" I heard this question a number of times during my research. However, I also heard that students who asked this question got very different answers depending on the mathematics course that they were taking. M 33 students heard answers that related to the prescribed program of studies, while AM 30 students heard answers relating to architecture, airlines, art, sound waves, and business. These answers seemed to provide the AM 30 students with a reason for studying mathematics, and therefore made it relevant to them. The findings of this research support the opinion that this is a significant step toward improving mathematics education in Alberta.

REFERENCES

Alberta Education. (1991). *Mathematics 23 – 33 – Programs of studies*. Edmonton, AB: Alberta Education.

Alberta Education. (1996). *Alberta program of studies for k-12 mathematics*. Edmonton, AB: Alberta Education.

Alberta Education. (1998). *Mathematics applied and pure programs – Programs of studies*. Edmonton, AB: Alberta Education.

Alberta Learning. (2002a). Applied Mathematics 30 diploma examination information bulletin. Edmonton, AB: Alberta Learning.

Alberta Learning. (2002b). *Mathematics 33 diploma examination information bulletin.* Edmonton, AB: Alberta Learning.

Alberta Learning. (2002c). Guide to education: ECS to grade 12. Retreived April 10, 2003 from http://www.learning.gov.ab.ca/educationguide/default.asp .

Apple, M. W. (1992). Do the standards go far enough? Power, policy and practice in mathematics education. *Journal for Research in Mathematics Education*, 23(1), 412 – 431.

Ames, C. (1992). Classrooms: Goals, structures, and student motivation. *Journal of Educational Pyschology*, 84, 261 – 271.

Battista, M. (1999). The mathematical miseducation of america's youth: Ignoring research and scientific study in education. *Phi Delta Kappan,* 80(6), 424 – 433.

Baroody, A. J., & Ginsburg, H. P. (1990). Children's mathematical learning: A cognitive view. In R. B. Davis, Carolyn Maher, & Nel Noddings (Eds.), *Journal for research in mathematics education, Monograph number 4*. (pp. 51 – 64). VA: NCTM.

Bauersfeld, H. (1995). The structuring of the structures: Development and function of mathematizing as a social practice. In L. P. Steffe & Jerry Gale (Eds.), *Constructivism in education* (pp. 137 – 158). New Jersey: Lawrence Erlbaum Associates.

Bennett, B., & Rolheiser, C. (2001). *Beyond Monet – The artful science of instructional integrataion.* Toronto: Bookation Inc.

Blumenfeld, P. (1992). Classroom learning and motivation: Clarifying and expanding goal theory. *Journal of Educational Psychology*, 84, 272 – 281.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education,* 29 (1), 41 – 62.

Brooks, J., & Brooks, M. (1993). *In search of understanding – The case for constructivist classrooms*. Virginia: Association for Supervision and Curriculum Development.

Brown, C. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209–242). New York: Macmillan.

Butterworth, B. (1999). *What counts, How every brain is hardwired for math*. New York: The Free Press.

Carraher, T. N., Schliemann, A. D., & Carraher, D. W. (1988). Mathematical concepts in everyday life. In G.B. Saxe & M. Gearhart (Eds.), *Children's mathematics* (pp. 71 – 88). San Francisco: Jossey-Bass.

Cobb, G. W. (1997). Mere literacy is not enough. In L. A. Steen (Ed.), *Why numbers count* (pp. 45 – 58). NY : The College Board.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representaional view of mind in mathematics education. *Journal for Research in Mathematics Education,* 23 (1), 2 – 33.

Confrey, J. (1990). What constructivism implies for teaching. In R. B. Davis, C. Maher, & N. Noddings (Eds.), *Journal for research in mathematics education, Monograph number 4* (pp. 107 – 124). VA: NCTM.

Confrey, J. (1994). A theory of intellectual development. *For the learning of mathematics*, 14 (3), 2 – 8.

Covington, M. V. (1992). *Making the grade: A self-worth perspective on motivation and school reform.* Cambridge, England: Cambridge University Press.

Creswell, J. (1998). *Qualitative inquiry and research design – Choosing among five traditions.* Thousand Oaks, California: Sage Publications.

Cumming, J. (1996). *From alienation to engagement: Opportunities for reform in the middle years of schooling, Column 3 teacher action.* Australia: Autstralian Curriculum Studies Association.

Davis, B. (1995). Why teach mathematics? Mathematics education and enactivist theory. *For the Learning of Mathematics*, 15 (2), 2 – 9.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28 (3), 355 – 376.

Davis, B., Kieren, T., & Sumara, D. (1996). Cognition, co-emergence and curriculum. *Journal of Curriculum Studies*, 28 (2), 151 – 169.

Davis, B., Sumara, D., & Luce-Kaplan, R. (2000). *Engaging minds – Learning and teaching in a complex world.* New Jersey: Lawrence Erlbaum Associates.

Denning, P. J. (1997). Quantitative practices. In L. A. Steen (Ed.), *Why numbers count* (pp. 106 – 117). NY: The College Board.

Dossey, J. (1997). National indicators of quantitative literacy. In L. A. Steen (Ed.), *Why numbers count* (pp. 34 – 44). NY: The College Board.

Dweck, C. S. (1986). Motivational processes affecting learning. *American Psychology*, 41, 1040 – 1048.

Eccles, J., Adler, T., & Meece, J. (1984). Sex differences in achievement: A test of alternate theories. *Journal of Personality and Social Psychology*, 46 (1), 26 – 43.

Elliot, E. S., & Dweck, C. S. (1988). Goals: An approach to motivation and achievement. *Journal of Personality and Social Psychology*, 54, 5 – 12.

Ellis, W. Jr. (2001). Numerical common sense for all. In L. A. Steen (Ed.), *Mathematics and democracy – The case for quantitative literacy* (pp. 61 – 65). National Council on Education and the Disciplines.

Ernest, P. (1991). *The philosophy of mathematics education*. London: Falmer.

Eyres, V. (1997). Steering a middle course. *EQ Australia*, Issue One Autumn, 8 – 9.

Fennema, E., & Sherman, J. (1978). Sex-related differences in mathematics achievement and related factors: A further study. *Journal for Research in Mathematics Education*, 9, 189 – 203.

Forman, S. L. & Steen, L. A. (1995). Mathematics for work and life. In I. M. Carl (Ed.), *Prospects for school mathematics* (pp. 219 – 241). Reston VA: NCTM.

Frank, M. L. (1988). Problem solving and mathematical beliefs. *Arithmetic Teacher*, 35, 32 – 34.

Frank, M. L. (1990). *The effects of problems solving software on the problem solving ability of secondary school students*. (Doctoral dissertation, University of Illinois, Urbana). Disertation Abstracts International, 48, 1804A.

Frankel, J., & Wallen, N. (2000). *How to design and evaluate research in education (4th ed.).* New York: McGraw Hill Higher Education.

Funkhouser, C. (1993). The influence of problem-solving software on student attitudes about mathematics. *Journal of Research in Computing in Education*, 25 (3), 339 – 346.

Gal, I. (1997). Numeracy: Imperatives of a forgotten goal. In L. A. Steen (Ed.), *Why numbers count* (pp. 36 – 44). NY: The College Board.

Ginsburg, L., & Gal, I. (1995). Linking informal knowledge and formal skills: The case of percents. In D. T. Owens, M. K. Reed, & G. M. Millsaps (Eds.), *Proceedings of the 17th annual meeting of the North American chapter of the international group for psychology of mathematics education.* Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Greenes, C. (1995). Mathematics learning and knowing: A cognitive process. *Journal of Education*, 177 (1), 85 – 97.

Haladyna, T., Shaughnessy, J., & Shaughnessy, J. M. (1983). A causal model of attitude toward mathematics. *Journal for Research in Mathematics Education,* 14, 19 – 29.

Hart. L. E. (1989). Describing the affective domain: Saying what we mean. In D. B. McLeod and V. M. Adams (Eds.), *Affect and mathematical problem solving, A new perspective* (pp. 37 – 45). New York: Springer – Verlag.

Higgins, K. M. (1997). The effect of year-long instruction in mathematical problem solving on middle-school students' attitudes, beliefs, and abilities. *Journal of Experimental Education*, 66 (1), 5 – 30.

Hughes-Hallet, D. (2001). Achieving numeracy: The challenge of implementation. In L. A. Steen (Ed.), *Mathematics and democracy – The case for quantitative literacy* (pp. 93 – 98). National Council on Education and the Disciplines.

Kieren, T. (1998). Teaching mathematics (in the middle): Enactivist view on learning and teaching mathematics. *Unpublished essay*. University of Alberta, Edmonton, Canada.

Kloosterman, P. (1998). How hard do you work in mathematics? Motivational profiles of six high school students. Paper Presented at the Annual Meeting of the AERA, San Diego, April 17, 1998. Retreived December 21, 2001 from http://www.indiana.edu/~pwkwww/AERA98.html.

Kolata, G. (1997). Understanding the news. In L. A. Steen (Ed.), *Why numbers count.* (pp. 23 – 29). NY: The College Board.

Koller, O., Baumert, J., & Schnabel, K. (2001). Does interest matter? The relationship between academic interest and achievement in mathematics. *The Journal for Research in Mathematics Education*, 32 (5), 448 – 470.

Krapp, A., Hidi, S., & Reinninger, K.A. (1992). *The role of interest in learning and development.* Hillsdale NJ: Erlbaum.

Licht, B. (1993). Achivement-related beliefs in children with learning disabilites: Impact on motivation and strategic learning. In L. Meltzer (Ed.), *Strategy assessment and instruction for students with learning disabilities: From theory to practice.* (pp. 195 – 220). Austin, TX: Pro-Ed.

Licht, B., & Dweck, C. S. (1984). Determinants of academic achievement: The interaction of children's achievement orientations with skill area. *Developmental Psychology*, 20, 628 – 636.

Ma, X. & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education*, 28 (1), 26 – 47.

Maher, C. J. (1982). *Teaching models in education of the gifted.* Rockville, MD: Aspen Systems Corp.

Manouchehri, A. & Goodman, H. (1998). Mathematics curriculum reform and teachers: Understanding the connections. *Journal of Educational Research*, 92 (1), 27 – 42.

Maturana, H. & Varela, F. (1987). *The tree of knowledge: The biological roots of human understanding.* Boston: Shambhala.

McCabe, K. (2000). *The case of the implementation of the western canada protocol in mathematics in Alberta.* Unpublished master's thesis, University of Alberta, Edmonton, Canada.

McLeod, D.B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of Research on mathematics teaching and learning* (pp. 575 – 596). New York: Maxwell Macmillan.

Merriam, S. (1988). *Case study research in education – A qualitative approach.* San Francisco: Jossey-Bass Inc.

Montague, M. (1997). Student perception, mathematical problem solving, and learning disabilities. *Remedial and Special Education*, 18 (1), 46 – 53.

Mullins, I., Martin, M., Gonzalez, E., Gregory, K., Garden., O'Connor, K., et al. (2000). *TIMMS 1999 international mathematics report.* Boston: Lynch School of Education.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1998). Teacher qualifications tied to students' achievement. *NCTM news bulletin*. 34 (8), pp 2 – 3.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics.* USA: National Academy Press.

Neale, D. C. (1969). The role of attitudes in learning mathematics. *Arithmetic Teacher*, 16, 631–640.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. Maher, & N. Noddings (Eds.), *Journal for research in mathematics education, Monograph number 4*. (pp. 7 – 18). VA: NCTM.

Noddings, N. (1994). Does everybody count? Reflections on reform in school mathematics. *Journal of Mathematical Behavior,* 13, 89 – 104.

Panitz, T. (1997). *Why more teachers do not use cooperative learning techniques.* Retrieved March 17, 2000, from http://home.capecod.net/~tpanitz/tedsarticles/whyfewclusers.htm .

Paulos, J. A. (1988). *Innumeracy*. New York: Vintage Books.

Pedro, J., Wolleat, P., Fennema, E., & Becker, A. (1981). Election of high school mathematics by females and males: Attributions and attitudes. *American Educational Research Journal*, 18 (2), 207 – 218.

Porter, T. (1997). The triumph of numbers: Civic implications of quantitative literacy. In L. A. Steen (Ed.), *Why numbers count* (pp. 1 – 10). The College Board: NY.

Resnick, L. B. (1995). Inventing arithmetic: Making children's intuition work at school. In C.A. Nelson (Ed.), *Basic and applied perspectives on learning, cognition, and development*. (pp 75 – 101). Mahwah, NJ: Lawrence Erlbaum Associates.

Resnick, L.B. (1988). Treating mathematics as an ill-structured discipline. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem-solving* (pp. 32 – 60). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schiefele, U. (1992). Topic interest and level of text comprehension. In K. A. Reinninger, S. Hidi, & A. Krapp (Eds.), *The role of interest in learning and development* (pp 151 – 182). Hillsdale NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1988) When good teaching leads to bad results. The disasters of "well-taught" mathematics courses. *Educational Psychologist*, 23, 145 – 166.

Shifter, D. (1996). A constructivist perspective on teaching and learning mathematics. *Phi Delta Kappan,* 78, 492 – 499.

Silver, E., Kilpatrick, J., and Schlesinger, B. (1990). *Thinking through mathematics*. New York: The College Board.

Singley, M. K., & Anderson, J. R. (1989). *The transfer of cognitive skill*. Cambrigde, Mass.: Harvard University Press.

Skemp, R. (1987). *The psychology of Llearning mathematics*. Hillsdale NJ: Lawrence Erlbaum Associates.

Stake, R. (1995). *The art of case study research*. Thousand Oakes, CA: Sage Publications.

Stake, R., & Trumbull, D. (1982). Naturalistic generalizations. *Review Journal of Philosophy and Social Science,* 7 (1), 1 – 12.

Steen, L. A. (1989). Teaching mathematics for tomorrow's world. *Educational Leadership,* 47 (1), 18 – 22.

Steen, L. A. (Ed.) (2001). *Mathematics and democracy – The case for quantitative literacy*. National Council on Education and the Disciplines.

Steffe, L., & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education*, 25 (6), 711 – 733.

Steinback, M., & Gwizdala, J. (1995) Gender differences in mathematics; Attitudes of secondary students. *School Science of Mathematics*, 95(1), 36 – 42.

Sternberg, R. J. (1985). *Beyond IQ: A triarchic theory of human intelligence*. New York: Cambridge.

Tobias, S. (1987). *Succeed with math: Every student's guide to conquering math anxiety*. New York: College Entrance Examination Board.

Travers, R. M. W. (1978). *Children's interest*. Kalamazoo, MI: Western Michigan University, College of Education.

Turner, J. C.; Cox, K. E.; Dicinto, M.; Meyer, D. K.; Logan, C. & Thomas, C. T. (1998). Creating contexts for involvement in mathematics. *Journal of Educational Psychology*, 90, 730 – 745.

University of Alberta. (2003). *2003/2004 University of Alberta Calendar*. Retreived March 30, 2003 from http://www.registrar.ualberta.ca/calendar/Admission/chart2.html.

Unland, K. (2000, December 6). Alta. students rank near top in global math, science tests. *The Edmonton Journal*, p. A1.

Usiskin, Z. (2001). Quantitative literacy for the next generation. In L. A. Steen (Ed.), *Mathematics and democracy – The case for quantitative literacy.* (pp. 79 – 86). National Council on Education and the Disciplines.

Utsumi, M. C., and Mendes, C. R. (2000). Researching the attitudes towards mathematics in basic education. *Educational Psychology*, 20 (2), 237 – 243.

Varela, F., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience.* Cambridge, MA: MIT Press.

Von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davis, C. Maher, & N. Noddings (Eds.), *Journal for research in mathematics education, Monograph number 4* (pp. 19 – 30). NCTM: VA.

Von Glasersfeld, E. (1995a). A constructivist approach to teaching. In L. Steffe, & J. Gle (Eds..), *Constructivism in education* (pp. 3- 16). Hillsdale NJ: Lawrence Erlbaum Associates.

Von Glasersfeld, E. (1995b). *Radical constructivism, A way of knowing and learning.* Falmer Press: London.

Von Glasersfeld, E. (1995c). Sensory experience, abstraction, and teaching. In L. Steffe, & J. Gle (Eds.), *Constructivism in education* (pp. 369 – 383). Hillsdale NJ: Lawrence Erlbaum Associates.

Walker, R. (1980). The conduct of educational case studies: Ethics, theory and procedures. In W.B. Dockerell & D. Hamilton, (Eds.), *Rethinking educational research* (pp. 30 – 63). London: Hodder & Stoughton.

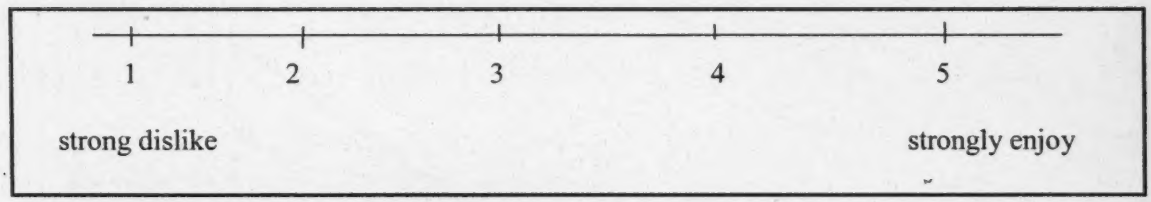
Yasutake, D., & Bryan, T. (1995). The influence of affect on the achievement and behaviour of students with learning disabilities. *Journal of Learning Disabilities*, 28, 329 – 334.

Weiner, B. (1984). Principles for a theory of student motivation and their application within an attributional framework. In R. E. Ames & C. Ames (Eds.), *Research on motivation in education* [Volume 1: Student Motivation] (pp. 15-38). Orlando, FL: Academic Press.

Western Canadian Protocol for Collaboration in Basic Education. (1996). *The common curriculum framework: K – 12 mathematics.* Edmonton, Alberta: Alberta Learning.

APPENDIX: QUESTIONNAIRE

1. Indicate your level of enjoyment of mathematics by circling one number between 1 and 5 — 1 being a strong dislike of mathematics and 5 being a strong enjoyment of mathematics.



If you circled a 3 or less, please answer this question:
Do you remember ever enjoying mathematics? Describe the activities or events that occurred during a math class that you enjoyed.

_____
_____
_____
_____

If you circled 4 or 5, please answer this question:
Do you remember ever not enjoying mathematics? Describe the activities or events that occurred during a math class that you did not enjoy.

_____
_____
_____
_____

2. What kinds or mathematics activities or lessons do you enjoy the most?

_____
_____
_____
_____

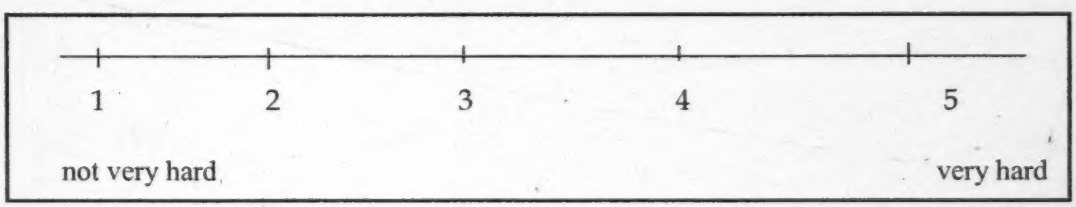
3. What kinds of mathematics activities or lessons do you enjoy the least?

4. Describe one thing you have done in this math class that you found interesting, or that sticks in your mind. Discuss why you remember this situation.

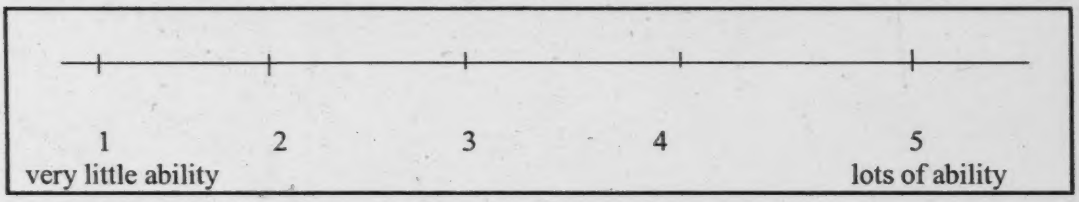
5. Describe one thing you have done in this math class that you did not find interesting. Discuss why you remember this situation.

6. How much do you participate in mathematics class? Why?

7. How hard do you work in mathematics class? Rate your work effort from 1 to 5, 1 being not very hard at all, and 5 being work very hard.



8. How much mathematical ability do you posses? Rate your ability from 1 to 5, 1 being not very much mathematical ability, and 5 being a great deal of mathematical ability.



9. What topics have you studied in math class that you will use in later life?

_____
_____
_____
_____

10. Explain why mathematics is, or is not important to you.

_____
_____
_____
_____

11. Explain why mathematics is, or is not important to society.

_____
_____
_____
_____

12. Have you ever used something you learned in math class in any other class? If so, describe what it was.

_____
_____
_____
_____

13. Explain why you would, or would not like to take any more mathematics courses.

14. You have demonstrated success in mathematics class by completing the prerequisite courses to Math 33/Applied Math 30. If you had to choose between ability and hard work, to what would you attribute your success in math class?

15. Why are you enrolled in Applied Mathematics 30/Mathematics 33 rather than a different mathematics class?